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*Price One Shilling and Sixpence.*

KEY  
TO THE  
RUDIMENTARY TREATISE ON  
ARITHMETIC:

CONTAINING SOLUTIONS IN FULL TO  
THE EXERCISES.

By J. R. YOUNG,  
Late Professor of Mathematics, Belfast College.

LONDON: JOHN WEALE. 1853.

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1780



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ON ARITHMETIC.



# K E Y

TO THE

RUDIMENTARY TREATISE ON ARITHMETIC:

CONTAINING SOLUTIONS IN FULL TO

THE EXERCISES,

TOGETHER WITH COMMENTS, EXPLANATIONS, AND IMPROVED PROCESSES,  
FOR THE USE OF TEACHERS AND UNASSISTED LEARNERS.

BY J. R. YOUNG,

Late Professor of Mathematics, Belfast College.

SECOND EDITION.

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## PREFACE.

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THIS book has been composed with the view of supplying an acceptable accompaniment to the "Rudimentary Treatise on Arithmetic." To the Teacher it will be found to furnish all that he is likely to seek in a Key; while to the otherwise unassisted Learner it will afford that kind of explanatory aid so much needed by those who have not the advantage of oral instruction.

My object having been to teach the principles of Arithmetic to those who know nothing about it, I have, as well in this as in the former book, kept the requirements of the solitary and self-dependent student constantly present to my mind—and I have written more for him than for the experienced Instructor. It is possible, therefore, that a schoolmaster who may use this book only for the ordinary purposes of a Key, may find in it an amount of explanation and minute detail unnecessary for his single object; but I have no fear that any enlightened Teacher will consider this a useless redundancy: all are interested in the simplifications of science; and I shall consider it a compliment,

rather than a censure, if this little work should be characterised, as the former volume was characterised in a recent Review, as "too easy."

Except in the way of simplification of the methods of operation, and of a more sound and logical establishment of first principles, little or nothing really remains to be done in so hackneyed a subject as Arithmetic; and from my conviction of this, the slight improvement I have introduced at page 61, in the process for the Common Measure, was as little anticipated by me, when I commenced this Key, as it will, no doubt, be unexpected by those who consult it.

The Arithmetic and Key together will form a volume of about 350 pages: the paper is good, and the print of the most expensive kind: the charge to the purchaser is *three shillings*.\* No one will say that the enterprising publisher has not done his part as to *quantity*; and if I have failed in mine as respects the more important item of *quality*, I can only say that the failure must arise from sheer inability, and not from any want of painstaking endeavour.

J. R. YOUNG.

\* The Arithmetic alone may be had bound strongly in cloth for 2s.

LONDON, Jan. 1, 1858.

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# KEY

TO

## THE RUDIMENTARY TREATISE ON ARITHMETIC.

*Exercises in Addition, pages 7, 8, 9.*

$$\begin{array}{r} (1) \quad 342 \\ 165 \\ 34 \\ \hline 541 \end{array}$$

*Carryings* 11

$$\begin{array}{r} (2) \quad 273 \\ 87 \\ 49 \\ \hline 409 \\ 21 \end{array}$$

$$\begin{array}{r} (3) \quad 2860 \\ 1723 \\ 41 \\ 17 \\ \hline 4641 \\ 111 \end{array}$$

$$\begin{array}{r} (4) \quad 5693 \\ 6297 \\ 482 \\ 13 \\ \hline 12485 \end{array}$$

*Carryings* 121

$$\begin{array}{r} (5) \quad 17341 \\ 20061 \\ 9203 \\ 510 \\ \hline 47115 \\ 111 \end{array}$$

$$\begin{array}{r} (6) \quad 35208 \\ 762070 \\ 187 \\ 62 \\ \hline 797527 \\ 21 \end{array}$$

$$\begin{array}{r} (7) \quad 7407003 \\ 169205 \\ 4853 \\ 382 \\ 79 \\ \hline 7581522 \end{array}$$

*Carryings* 2122

$$\begin{array}{r} (8) \quad 2014 \\ 7035 \\ 101009 \\ 17048 \\ 201 \\ \hline 127297 \\ 1 \quad 2 \end{array}$$

$$\begin{array}{r} (9) \quad 1120 \\ 418 \\ 2398 \\ 3227 \\ 2869 \\ 1780 \\ \hline 11812 \text{ yds.} \\ 233 \end{array}$$

## 2 KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

<i>Census, 1841.</i>		<i>Census, 1851.</i>		
(10)	15911725	17922768	(11)	107815
	2628957	2870784		109915
	124079	142916		109760
<i>Population</i>	<u>18664761</u>	<u>20936468</u>		90813
<i>Carryings</i>	1 1112	111 211		46913
				<u>53061</u>
				<i>Visits to the Exhibition in the last week</i> 518277
				<i>Carryings</i> 23411
(12)	27	734814	(13)	212124
	25	1133114		17127
	27	1314176		70247
	26	1023435		214612
	26	1155240		15154
	13	808237		<u>51083</u>
	<u>144 days</u>	<u>6169016</u>	<i>Visits altogether.</i>	<i>Em. in 2 yrs.</i> 580347 <i>persons</i>
<i>Carryings</i>	3	212222		<u>12122</u>
	<i>Newspapers.</i>			<i>Advertisements.</i>
(14)	159		(15)	891650
	222			875631
	110			249141
	102			<u>236128</u>
	<u>Total</u> 593		<u>Total</u>	2252550
<i>Carryings</i>	1			<u>22111</u>



### *Exercises in Subtraction, pages 12, 13, 14.*

(1)	846	2865	(2)	72098
	375	1237		<u>36207</u>
<i>Remainder</i>	<u>471</u>	<u>1628</u>		<u>35891</u>
<i>Carryings</i>	1	1		11
(3)	18097	1001251	(4)	111002
	7992	300043		<u>7053</u>
<i>Rem.</i>	<u>10105</u>	<u>701208</u>		<u>103949</u>
<i>Carryings</i>	1	1		1111

(5) $\begin{array}{r} 22005 \\ 13117 \\ \hline \end{array}$	(6) $\begin{array}{r} 3000111 \\ 1302042 \\ \hline \end{array}$	(7) $\begin{array}{r} 1851 \\ 1769 \\ \hline \end{array}$
<i>Remainder</i> 8888	1698069	<i>Age in 1851, 82 yrs.</i>
<i>Carryings</i> 1111	111 11	11
(8) $\begin{array}{r} 8175124 \\ 6515794 \\ \hline \end{array}$	(9) $\begin{array}{r} 7767401 \\ 6801827 \\ \hline \end{array}$	(10) $\begin{array}{r} 20936468 \\ 18664761 \\ \hline \end{array}$
<i>Decr.</i> 1659330	<i>Incr.</i> 965574	<i>Incr.</i> 2271707
<i>Carryings</i> 1 111	1 111	1 1 1
(11) $\begin{array}{r} 19507 \\ 8615 \\ \hline \end{array}$	(12) $\begin{array}{r} 1098863 \\ 221119 \\ \hline \end{array}$	(13) $\begin{array}{r} 2264684 \\ 1460785 \\ \hline \end{array}$
10892 *	877744	803899 <i>pounds</i>
<i>Carryings</i> 11	1	1 111
(14) $\begin{array}{r} 41087919 \\ 31766503 \\ \hline \end{array}$	(15) $\begin{array}{r} 6217 \\ 2135 \\ \hline \end{array}$	(16) $\begin{array}{r} 17922768 \\ 142916 \\ \hline \end{array}$
<i>Incr.</i> 9321416	8352	18065684
<i>Carryings</i> 11	<i>Sub.</i> 8149	20936468
	<i>Increase</i> 203 <i>pounds.</i>	<i>Sub.</i> 18065684
	<i>Population of Scotland</i> 2870784	
(17) $\begin{array}{r} 550236 \\ 62115 \\ 57903 \\ \hline \end{array}$	$\begin{array}{r} 22481339 \\ 670254 \\ \hline \end{array}$	(18) $\begin{array}{r} 578159 \\ 295158 \\ \hline \end{array}$
£670254	£21811085 <i>nett amount.</i>	<i>Born</i> 283001; <i>died</i> 219052
(19) $\begin{array}{r} 365 \\ 2041 \\ 109 \\ \hline \end{array}$	(20) $\begin{array}{r} 112104 \\ 3820 \\ 3268 \\ \hline \end{array}$	(21) $\begin{array}{r} 462873 \\ 5962 \\ 304 \\ \hline \end{array}$
2515	119192	19871
12646	119192	489010
2515	<i>Diff.</i> 0	904549
<i>Diff.</i> 10131		<i>Diff.</i> 415539

*Exercises in Multiplication, pages 18, 19.*

(1) $\begin{array}{r} 342 \\ 3 \\ \hline 1026 \end{array}$	(2) $\begin{array}{r} 4761 \\ 4 \\ \hline 19044 \end{array}$	(3) $\begin{array}{r} 7065 \\ 5 \\ \hline 35325 \end{array}$	(4) $\begin{array}{r} 80724 \\ 6 \\ \hline 484344 \end{array}$
<i>Carryings</i> 1	32	32	412

(5) $\begin{array}{r} 1139509 \\ 7 \\ \hline 7976563 \end{array}$	(6) $\begin{array}{r} 273 \\ 12 \\ \hline 3276 \end{array}$	(7) $\begin{array}{r} 75200564 \\ 8 \\ \hline 601604512 \end{array}$
<i>Carryings</i> 263 6	83	41 453

(8) $\begin{array}{r} 9264073128 \\ 11 \\ \hline 101904804408 \end{array}$	(9) $\begin{array}{r} 650098203470 \\ 12 \\ \hline 7801178441640 \end{array}$	(10) $\begin{array}{r} 8615 \\ 2 \\ \hline 17230 \end{array}$	$\begin{array}{r} 10892 \\ 3 \\ \hline 32676 \\ 17230 \\ \hline 49906gs. \end{array}$
<i>Carry.</i> 274 83138	6 11192 458		

(11) $\begin{array}{r} 89319 \\ 6 \\ \hline 535914 \end{array}$	(12) 365000 Any number becomes multiplied by 1000 when three 0's are put against it on the right.—See Art. 22, page 18.	(13) <i>Add.</i> $\begin{array}{r} \{ 44512 \\ 9327 \\ \hline 53839 \\ 5 \\ \hline 269195s. \end{array}$
---	--	--

(14) $\begin{array}{r} 1125 \\ 2 \\ \hline 2250ft. \end{array}$	$\begin{array}{r} 1125 \\ 7 \\ \hline 7875feet. \end{array}$	(15) $\begin{array}{r} 1125 \\ 11 \\ \hline 12375feet. \end{array}$	(16) $\begin{array}{r} 7239962 \\ 12 \\ \hline 86879544letters. \end{array}$
---	--	---	--

*Pages 24, 25.*

(1) $\begin{array}{r} 463 \\ 247 \\ \hline 3241 \\ 1852 \\ 926 \\ \hline 114361 \end{array}$	(2) $\begin{array}{r} 789 \\ 674 \\ \hline 3156 \\ 5523 \\ 4734 \\ \hline 531786 \end{array}$	(3) $\begin{array}{r} 2345 \\ 896 \\ \hline 14070 \\ 21105 \\ 18760 \\ \hline 2101120 \end{array}$	(4) $\begin{array}{r} 67082 \\ 7034 \\ \hline 268328 \\ 201246 \\ 469574 \\ \hline 471854788 \end{array}$
--	---	--	---



(5)	82060 5831 <hr/> 82060 24618 65648 41030 <hr/> 478491860	(6)	34728 65900 <hr/> 31255200 173640 208368 <hr/> 2288575200	(7)	807900 64300 <hr/> 242370000 32316 48474 <hr/> 51947970000
-----	--	-----	--	-----	---

(8)	250978 64007 <hr/> 1756846 1003912 1505868 <hr/> 16064348846	(9)	76830450 2001650 <hr/> 3841522500 46098270 7683045 <hr/> 15368090 <hr/> 153787670242500
-----	---	-----	---

(10)	3456789 9876543 <hr/> 10370367 13827156 17283945 20740734 24197523 27654312 31111101 <hr/> 34141125200427	(11)	372 583 <hr/> 1116 2976 1860 <hr/> 216876 261 <hr/> 216876 1301256 433752 <hr/> 56604636
------	--	------	--

The method of proving multiplication by *casting out the nines* is explained at p. 24 of the Arithmetic ; if we apply this method to Ex. 10, we shall find that, after casting out the 9's from the multiplicand there will remain 6, and after casting out the 9's from the multiplier, that there will remain also 6. The product of these remainders is 36, out of which if the 9's be cast, there will remain *nothing*; consequently, if the work be correct, *nothing* will remain after casting the 9's out of the product, and such is found to be the case.



It is usual to write the two remainders given by multiplicand and multiplier, one on each side of a cross, as in the margin, then, having rejected the 9's from the product of the numbers thus written, to put the remainder above, and lastly, to put the remainder arising from the product underneath ; in the present case these remainders are each 0.

When we have three factors and only the final product of all, we may apply a similar method for trying the correctness of this final product; thus, taking the three factors, 372, 583, 261, of Ex. 11, we may proceed as above with the first two, putting 3 on the left, and 7 on the right of the cross: the product of these is 21, which gives 3 for remainder. Now, instead of putting this 3 *above* the cross, as we should do if only *two* factors were concerned, let it be brought to the *left*, and let the remainder, 0, given by the third factor, namely, 261, be placed on the *right*; then, rejecting the 9's from the product of these two new numbers, place the remainder, 0, above the cross, and the remainder arising from the final product, below, as in the margin. And we may proceed in a similar way, be the factors ever so numerous, the upper part of the cross being left vacant till after the final factor has been disposed of; if there be four factors there will be *three* numbers on each side of the cross; the extreme numbers, that is, the two farthest from the cross, one on each side, are to be multiplied together, and the remainder given by the product is to be placed *above*, which remainder must be the same as that given by the product of the factors, and placed below, if that product be correct. It is scarcely necessary to observe, that when any one of the factors gives a remainder 0, the product of all must give a remainder 0; and, more generally, if a 0 occur on either side of the cross, it will indicate that the product cannot give any remainder.\*



\* I do not know whether the extension of the method from two to any number of factors has been given before. As to the method itself, the learner must bear in mind that it does not furnish an absolutely unfailling test of the accuracy of a multiplication result. When such result is correct the condition is always satisfied, but it will also be satisfied if an error of 9 or of any *multiple* of 9, that is, of any *number of times* 9 occur in the product; if, however, an error of a different kind have been committed, the method will always give indication of it; so that when no error is indicated the product is *most likely* correct.

The general principle mentioned at Article 28 in the Arithmetic, and upon which this mode of proof is founded, suggests a curious property of numbers, namely, that any two numbers consisting of the same figures, however differently arranged, will furnish equal remainders when each is divided by 9, and, *consequently*, that the difference of two such numbers will always be divisible by 9; thus, 3742 and 2734 are two numbers formed by the same figures; the difference between them is 1008, which is 112 times 9; again, the difference between 374268 and 463287 is 89019, which is 9891 times 9; and so in all cases. When the learner has reached short division, he may frame for himself some easy and interesting exercises upon this property, such, for instance, as:—Find what multiple of 9 subtracted from 4112 will reverse the digits. Find what multiple of 9, 3627 must be subtracted from in order that the remainder may have the same digits, but in reverse order; and so on.

(12)	$\begin{array}{r} 168 \\ 18 \\ \hline 1344 \\ 168 \\ \hline 3024 \text{ miles.} \end{array}$	(13)	$\begin{array}{r} 47484134 \\ 16 \\ \hline 284904804 \\ 47484134 \\ \hline 759746144 \text{ miles.} \end{array}$	(14)	$\begin{array}{r} 192500 \\ 493 \\ \hline 577500 \\ 17325 \\ \hline 7700 \\ \hline 94902500 \text{ miles} \end{array}$
------	--	------	--	------	--

(15)	$\begin{array}{r} \text{Add. } \left\{ \begin{array}{l} 31766503 \\ 41087919 \\ \hline 72854422 \\ 17 \\ \hline 509980954 \\ 72854422 \\ \hline 1238525174 \text{ miles.} \end{array} \right. \end{array}$	(16)	$\begin{array}{r} 2018 \\ 212 \text{ or multiplying} \\ \hline 4036 \text{ first by the 12} \\ 2018 \text{ and then by} \\ 4036 \text{ the 2.} \\ \hline 427816 \text{ pounds.} \end{array}$	$\begin{array}{r} 2018 \\ 212 \\ \hline 24216 \\ 4036 \\ \hline 427816 \text{ pounds} \end{array}$
------	--	------	--	--

(17)	$\begin{array}{r} 1851 \\ 271 \\ \hline 1851 \\ 12957 \\ 3702 \\ \hline 501621 \end{array}$	$\begin{array}{r} 505107 \text{ (Ex. 8, p. 4.)} \\ \text{Sub. } 501621 \\ \hline 3486 \text{ difference.} \end{array}$
------	---	--

(18)	$\begin{array}{r} 2273 \\ 518 \\ \hline 18184 \\ 2273 \\ \hline 11365 \\ \hline \text{Gross Earnings } 1177414 \text{ pounds.} \\ \text{Expense } 735257 \text{ pounds.} \\ \hline \text{Nett Earnings } 442157 \text{ pounds.} \end{array}$
------	--

~~~~~

*Exercises in Short Division, pages 29, 30.*

|     |                                                         |     |                                                            |     |                                                             |
|-----|---------------------------------------------------------|-----|------------------------------------------------------------|-----|-------------------------------------------------------------|
| (1) | $\begin{array}{r} 5)3724 \\ 744\frac{1}{2} \end{array}$ | (2) | $\begin{array}{r} 6)72081 \\ 12013\frac{1}{2} \end{array}$ | (3) | $\begin{array}{r} 7)109234 \\ 15604\frac{2}{7} \end{array}$ |
|-----|---------------------------------------------------------|-----|------------------------------------------------------------|-----|-------------------------------------------------------------|

# 8 KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

$$(4) \begin{array}{r} 8)2006383 \\ \underline{250797\frac{1}{2}} \end{array} \quad (5) \begin{array}{r} 9)52094100 \\ \underline{5788233\frac{1}{2}} \end{array} \quad (6) \begin{array}{r} 11)11380625 \\ \underline{1034602\frac{3}{4}} \end{array}$$

$$(7) \begin{array}{r} 12)10792039 \\ \underline{899336\frac{1}{2}} \end{array} \quad (8) \begin{array}{r} 4)265837 \\ \underline{66459\frac{1}{4}} \end{array} \quad (9) \begin{array}{r} 7)872371 \\ \underline{124624\frac{1}{2}} \end{array}$$

$$(10) \begin{array}{r} 5)34205 \\ \underline{6841} \\ \text{Add } 207 \\ \underline{7048} \end{array} \quad (9) \begin{array}{r} 1863 \\ \underline{207} \end{array} \quad (11) \begin{array}{r} 8)24603705 \\ \underline{3075463\frac{1}{2}} \\ \text{Sub. } 10293 \\ \underline{3065170\frac{1}{2}} \end{array} \quad 11)113223 \\ \underline{10293}$$

$$(12) \begin{array}{r} 6)518277 * \\ \underline{86379} \text{ persons.} \end{array} \quad (13) \begin{array}{r} 4)6515794 \dagger \\ \underline{1628948} \text{ persons.} \end{array} \quad (14) \begin{array}{r} 5)621865 \\ \underline{124373} \text{ pounds} \end{array}$$

$$(15) \begin{array}{r} 7)7239962 \\ \underline{1034280} \text{ letters.} \end{array} \quad (16) \begin{array}{r} 18870 \\ \underline{1226283} \\ 8)1245153 \ddagger \\ \underline{155644\frac{1}{2}} \text{ pounds.} \end{array}$$

## *Exercises in Long Division, pages 35, 36.*

$$(1) \begin{array}{r} 47)2463(52\frac{19}{47} \\ \underline{235} \\ 113 \\ \underline{94} \\ 19 \end{array} \quad (2) \begin{array}{r} 83)39072(470\frac{13}{83} \\ \underline{332} \\ 587 \\ \underline{581} \\ 62 \end{array} \quad (3) \begin{array}{r} 342)1197054(3500\frac{54}{342} \\ \underline{1026} \\ 1710 \\ \underline{1710} \\ 54 \end{array}$$

(4) By looking at the table of factors at page 179, you will find that the divisor, 576, is the product of the three factors, 12, 12, 4;

\* This number is the answer to Example 11, page 8, of the book; it is divided by 6, because there were 6 visiting days, and we want to find how many visits were paid, *on the average*, each day.

† As this number is the population of all the four provinces, the fourth part of it must be the *average* population of each.

‡ We divide by 8 because there are 8 quarters in two years.

so that the present Example may be worked either by long division or short division. Both methods are here given:—

*By Long Division.*

$$576 \overline{) 8264921} (14348 \frac{171}{576}$$

$$\begin{array}{r} 576 \\ \underline{2504} \\ 2304 \\ \underline{2009} \\ 1728 \\ \underline{2812} \\ 2304 \\ \underline{5081} \\ 4608 \\ \underline{473} \end{array}$$

*By Short Division.*

$$\begin{array}{r} 12 \overline{) 8264921} \\ 12 \overline{) 686743} \frac{5}{13} \\ 4 \overline{) 57395} \frac{11}{13} \\ \underline{14348} \frac{171}{576} \end{array}$$

$$(5) \quad 843 \overline{) 9460257} (11222 \frac{111}{843}$$

$$\begin{array}{r} 843 \\ \underline{1030} \\ 843 \\ \underline{1872} \\ 1686 \\ \underline{1865} \\ 1686 \\ \underline{1797} \\ 1686 \\ \underline{111} \end{array}$$

$$(6) \quad 2712 \overline{) 40627385} (14980 \frac{161}{2712}$$

$$\begin{array}{r} 2712 \\ \underline{13507} \\ 10848 \\ \underline{26593} \\ 24408 \\ \underline{21858} \\ 21696 \\ \underline{1625} \end{array}$$

$$(7) \quad 3046 \overline{) 7926400} (2602 \frac{708}{3046}$$

$$\begin{array}{r} 6092 \\ \underline{18344} \\ 18276 \\ \underline{6800} \\ 6092 \\ \underline{708} \end{array}$$

$$(8) \quad 4026 \overline{) 00817293.61} (203 \frac{161}{4026}$$

$$\begin{array}{r} 8052 \\ \underline{12093} \\ 12078 \\ \underline{1561} \end{array}$$

$$(9) \quad 7891 \overline{) 01769023.4} (224 \frac{14394}{7891}$$

$$\begin{array}{r} 15782 \\ \underline{19082} \\ 15782 \\ \underline{33003} \\ 31564 \\ \underline{14394} \end{array}$$

$$(10) \quad 80247 \overline{) 27506381} (342 \frac{1907}{80247}$$

$$\begin{array}{r} 240741 \\ \underline{343228} \\ 320988 \\ \underline{222401} \\ 160494 \\ \underline{61907} \end{array}$$

# 10 KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

$$\begin{array}{r}
 (11) \\
 26083 \overline{) 59264743} (2272 \overset{4167}{\underset{26083}{}} \\
 \underline{52166} \\
 70987 \\
 \underline{52166} \\
 188214 \\
 \underline{182581} \\
 56333 \\
 \underline{52166} \\
 4167
 \end{array}$$

$$\begin{array}{r}
 (12) \\
 34275 \overline{) 10792564 \cdot 01} (314 \overset{3091401}{\underset{3427500}{}} \\
 \underline{102825} \\
 51006 \\
 \underline{34275} \\
 167314 \\
 \underline{137100} \\
 3021401
 \end{array}$$

*By Long Division.*

$$\begin{array}{r}
 (13) \quad 264 \overline{) 498834} (1662 \text{ * pounds.} \\
 \underline{264} \\
 1748 \\
 \underline{1584} \\
 1643 \\
 \underline{1584} \\
 594 \\
 \underline{528} \\
 66
 \end{array}$$

*By Short Division.*

$$\begin{array}{r}
 11 \overline{) 438834} \\
 6 \overline{) 39894} \\
 4 \overline{) 6649} \\
 \underline{1662} \text{ * pounds.}
 \end{array}$$

$$\begin{array}{r}
 (14) \quad 26 \overline{) 100000} (3846 \\
 \underline{78} \\
 220 \\
 \underline{208} \\
 120 \\
 \underline{104} \\
 160 \\
 \underline{156} \\
 4
 \end{array}$$

$$\begin{array}{r}
 (15) \quad 1585 \overline{) 20936468} (13209 \\
 \underline{1585} \\
 5086 \\
 \underline{4755} \\
 3314 \\
 \underline{3170} \\
 14468 \\
 \underline{14265} \\
 203
 \end{array}$$

(16) To divide 20936468 by 1000, it is only necessary to cut off the last three figures; thus, 20936,468: for 20936 will be the quotient, and 468 the *remainder*. Consequently the number of blind persons is 20936; and, since the number of deaf and dumb is 13209, the difference between these numbers, viz., 7727, shows how many blind persons there are more than deaf and dumb persons in Great Britain.

\* The *complete* quotient, by long division, is  $1662\frac{26}{26083}$ , and the *complete* quotient, by short division, is  $1662\frac{1}{2}$ . The fractions differ only

$$\begin{array}{r}
 (17) \quad 344 \overline{) 27000} (78 \frac{1}{2} \\
 \underline{2408} \\
 2920 \\
 \underline{2752} \\
 168
 \end{array}$$

If the mountain be taken only 4 feet higher, the remainder, instead of 168, will be 172, which is exactly half of 344; hence the quotient is  $78 \frac{1}{2}$  very nearly.

(18) Since two copies make 1 inch, we must divide the number of copies by 2, in order to find the number of inches in the pile, and must then divide this number, namely, 150000, by 2424, as follows:—

$$\begin{array}{r}
 2) 300000 \\
 \underline{150000}
 \end{array}$$

$$\begin{array}{r}
 2424 \overline{) 150000} (62 \text{ nearly.} \\
 \underline{14544} \\
 4560
 \end{array}$$

If the 150000 inches were increased by only 288 inches, the quotient would be exactly 62; and to at least this height the catalogues would really have reached, because, even when pressed, each was more than half an inch thick.

#### SHORT METHODS OF MULTIPLYING AND DIVIDING IN PARTICULAR CASES.

Time and figures may often be saved in arithmetical operations by the exercise of a little reflection at the outset. The following are some Examples of this:—

##### *Multiplication.*

##### 1. Multiply 324 by 25.

Here, instead of actually multiplying by 25, it will be better to multiply by 4 times 25, and then to take a fourth part of the product. As 4 times 25 is 100, this product, written down without any work, is 32400; so that the only operation is this, namely:—

$$\begin{array}{r}
 4) 32400 \\
 \underline{\phantom{00}} \\
 8100
 \end{array}$$

##### 2. Multiply 67233 by 125.

It is easily seen here, after a little thought, that 8 times 125 is 1000; therefore the operation is reduced to this, namely:—

$$\begin{array}{r}
 8) 67233000 \\
 \underline{\phantom{000000}} \\
 8404125
 \end{array}$$

in appearance, for each is *one-fourth*. The latter fraction is 1, divided by 4; and 1 divided by 4 is the same as twice 1 divided by twice 4, or 10 times 1 divided by 10 times 4, or any number of times 1 divided by the same number of times 4. The first fraction is 66 times 1 divided by 66 times 4, and is therefore equal to the second.

§ If the divisor be multiplied by the quotient, taken without any fractional correction, and the remainder be added to the product, the

## 12 KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

### 3. Multiply 864 by 75.

If we multiply by 100, we shall multiply by a fourth part of 100 too much, so that a fourth part of the product must be subtracted; therefore the operation is as follows:—

$$\begin{array}{r} 4)86400 \\ \underline{21600} \\ 64800 \end{array}$$

Or, since 4 times 75 is 300, we may proceed thus:—

$$\begin{array}{r} 864 \\ 300 \\ \hline 4)259200 \\ \underline{64800} \end{array}$$

Again, when the multiplier terminates in 9, or in a row of 9's, we shall save figures by adding 1 to the last 9, which will convert each 9 into a 0; the multiplier will thus be simplified, and the product by it, which is of course wrong, will become corrected by subtracting from it *once* the multiplicand, as in the Example next following:—

### 4. Multiply 2687 by 799.

By adding 1 to the multiplier, it becomes 800; and if we use this instead of 799, the product will obviously be *once* 2687 too great:—

$$\begin{array}{r} 2687 \\ 800 \\ \hline 2149600 \\ \text{Subtract } 2687 \end{array}$$

Correct prod. 2146913

The following Example, worked on a similar principle, will also be readily understood by the learner:—

### 5. Multiply 4283 by 9970.

Add 30 to the multiplier, then the work will be as follows:—

$$\begin{array}{r} 42830000 \\ \text{Sub. } 128490 = 4283 \times 30 \end{array}$$

Correct pro. 42701510

Other means of simplifying the work of multiplication, under particular circumstances, are explained in the foot note, at page 119 of the Arithmetic.

## Division.

When one number is to be divided by another, the quotient will not be altered, though we multiply both dividend and divisor by any number we please. The following Examples will show that the work may sometimes be shortened by making use of this privilege:—

### 1. Divide 72568 by 125.

$$\begin{array}{r} 125 \quad 72568 \\ 8 \quad \quad 8 \\ \hline 1,000) \quad 580,544 (580 \frac{44}{1000} \end{array}$$

Here 8 times the dividend is divided by 8 times the divisor, and the quotient is the same as if once

the dividend were divided by once the divisor. The fraction in the quotient is 544 divided by 1000; both these numbers will divide by 8, so that the fraction may be more simply written  $\frac{68}{125}$ . And the fractional part of the quotient may always be simplified, in like cases, by dividing both top and bottom

result will be the dividend, if the operation be correct. Hence, regarding quotient and divisor as multiplier and multiplicand, and the dividend, diminished by the remainder, as product, we may apply the method of casting out the 9's to prove *Division*.



numbers by the common multiplier of the original dividend and divisor, which in the present Example is 8.

2. Divide 685293 by 75.

$$\begin{array}{r} 75 \quad 685293 \\ \underline{4} \quad \underline{4} \\ 3,00) \quad 27411,72 \\ \underline{9137 \frac{1}{2}} \end{array}$$

Here, instead of putting down the remainder 72, with the divisor 300 underneath, and then dividing each by 4, for the purpose of simplification, 72 is divided by 4 at once, and the original divisor, 75, placed underneath, which is the better way.

When the divisor consists of NINES only, there is a very simple and expeditious method of finding the quotient and remainder. It may be stated in a Rule as follows:—

RULE.—Cut off from the right of the dividend as many figures as there are nines in the divisor.

Write the figures which precede those cut off under the dividend, so that the last of them may be under the last figure of the dividend, which is, of course, the last of the figures cut off.

From this second row cut off the same number of figures as before, and write the figures that precede them underneath, like as in the former case, and you will have a third row; and proceed in this way till you are stopped for want of figures.

The figures cut off, when added up, will furnish the remainder; the figures retained, when added up, will give the quotient; but, observe, that whatever you carry from the remainder-columns to the first quotient-column, must be afterwards also added to the remainder itself.

|                 |                   |                     |
|-----------------|-------------------|---------------------|
| (1) 99)625,87   | (2) 99)59342,68   | (3) 999)758062,513  |
| 625             | 593,42            | 758,062             |
| 6               | 5,93              | 758                 |
| Quotient 632,18 | Quotient 59942,08 | Quotient 758821,333 |
| 1               | 2                 | 1                   |
| Remainder 19    | Remainder 10      | Remainder 334       |

In the first of the dividends above, there are 625 *hundreds* and 87 remainder; consequently, without at present reckoning this remainder, there must be 625 *ninety-nines*, and 625 *units* over. In these 625 *units* there are 6 *hundreds* and 25 remainder; therefore, disregarding the remainder, there must be in these *units* 6 *ninety-nines* and 6 *units* over. The sum of the *remainders* is one *hundred* and 18, and, as the one *hundred* is one 99 and 1 over, the true remainder is 19. In the next Example the remainders amount to 200 and 8, and the 200 being 2 *ninety-nines* and 2, the correct remainder is 10. Similar explanations apply to the third Example.

*Reduction of Quantities to Lower Denominations,*  
pages 47, 48.

| $\pounds$ | s.            | d. |                   | $\pounds$ | s.            | d. |                   | Days. |                              |
|-----------|---------------|----|-------------------|-----------|---------------|----|-------------------|-------|------------------------------|
| (1)       | 865           | 17 | 5                 | (2)       | 397           | 16 | $4\frac{1}{2}$    | (3)   | 365                          |
|           | 20            |    |                   |           | 20            |    |                   |       | 24                           |
|           | <u>17317</u>  |    | <i>shillings.</i> |           | <u>7956</u>   |    | <i>shillings.</i> |       | <u>1460</u>                  |
|           | 12            |    |                   |           | 12            |    |                   |       | 730                          |
|           | <u>207809</u> |    | <i>pence.</i>     |           | <u>95476</u>  |    | <i>pence.</i>     |       | <u>8760</u> <i>hours.</i>    |
|           |               |    |                   |           | 4             |    |                   |       | 60                           |
|           |               |    |                   |           | <u>381907</u> |    | <i>farthings.</i> |       | <u>325600</u> <i>minutes</i> |

In the first of these Examples the pounds are reduced to shillings by multiplying the *number* of pounds by 20, because there are 20 times as many shillings as pounds; the learner must be careful to remember, that the *pounds* themselves are not multiplied by 20, only the *number* of them; and that the product is only the *number* of shillings; this *number*, afterwards multiplied by 12, gives the *number* of pence, because there are 12 times as many pence as shillings. In like manner, in Ex. 3, it is the *number* 365 merely that is multiplied by 24, to get the *number* of hours; and this latter *number* is then multiplied by 60 to get the *number* of minutes. In all such cases of reduction *abstract numbers* only are employed in the multiplications.

| cwt. | qrs. | lbs. |      | oz. | dwt.  | grs. |      | yds. | ft.  | in. |     |
|------|------|------|------|-----|-------|------|------|------|------|-----|-----|
| (4)  | 5    | 3    | 18   | (5) | 73    | 17   | 11   | (6)  | 237  | 1   | 6   |
|      | 4    |      |      |     | 20    |      |      |      | 3    |     |     |
|      | 23   |      | qrs. |     | 1477  |      | dwt. |      | 712  |     | ft. |
|      | 28   |      |      |     | 24    |      |      |      | 12   |     |     |
|      | 192  |      |      |     | 5909  |      |      |      | 8550 |     | in. |
|      | 47   |      |      |     | 2955  |      |      |      |      |     |     |
|      | 662  |      | lbs. |     | 35459 |      | grs. |      |      |     |     |

  

| mi. | fur.                | per. | yds.   |   | acres. |                  | t.  | cwt. | qrs. | lbs. |    |
|-----|---------------------|------|--------|---|--------|------------------|-----|------|------|------|----|
| (7) | 47                  | 5    | 9      | 3 | (8)    | 7                | (9) | 3    | 13   | 2    | 22 |
|     | 8                   |      |        |   |        | 4                |     | 20   |      |      |    |
|     | 381                 |      | fur.   |   |        | 28               |     | 73   |      | cwt. |    |
|     | 40                  |      |        |   |        | 40               |     | 4    |      |      |    |
| 2)  | 15249               |      | per.   |   | 4)     | 1120             |     | 294  |      | qrs. |    |
|     | 5 $\frac{1}{2}$     |      |        |   |        | 30 $\frac{1}{4}$ |     | 28   |      |      |    |
|     | 76248               |      |        |   |        | 33600            |     | 2354 |      |      |    |
|     | 7624 $\frac{1}{2}$  |      |        |   |        | 280              |     | 590  |      |      |    |
|     | 83872 $\frac{1}{2}$ |      | yards. |   |        | 33880            |     | 8254 |      | lbs  |    |

|                     | <i>t.</i>           | <i>t.</i> <i>cwt.</i> <i>qrs.</i> <i>lbs.</i> |
|---------------------|---------------------|-----------------------------------------------|
| (10) 46 <i>bar.</i> | (11) 1523           | (12) 5 2 1 22                                 |
| <u>36</u>           | <u>20</u>           | <u>20</u>                                     |
| 276                 | 30460 <i>cwt.</i>   | 102 <i>cwt.</i>                               |
| <u>138</u>          | <u>112</u>          | <u>4</u>                                      |
| 1656 <i>gals.</i>   | 60920               | 409 <i>qrs.</i>                               |
| <u>4</u>            | <u>33506</u>        | <u>28</u>                                     |
| 6624 <i>qts.</i>    | 3411520 <i>lbs.</i> | 3274                                          |
|                     |                     | <u>820</u>                                    |
|                     |                     | <u>11474 lbs.</u>                             |

| <i>t.</i> <i>cwt.</i> <i>lbs.</i> | <i>t.</i>         |
|-----------------------------------|-------------------|
| (13) 192 17 16                    | (14) 35           |
| <u>20</u>                         | <u>20</u>         |
| 3857 <i>cwt.</i>                  | 700 <i>cwt.</i>   |
| <u>112</u>                        | <u>112</u>        |
| 7720                              | 78400 <i>lbs.</i> |
| <u>42428</u>                      | <u>16</u>         |
| 432000 <i>lbs.</i>                | 470400            |
|                                   | <u>784</u>        |

*weight in avoird. ounces* 1254400 *oz.*  
5

*value, at 5s. per oz., in shillings* 6272000 *s.*

(15) One quart, one pint, and one half-pint, make together 7 half-pints; therefore if the pipe, or 126 gallons, be reduced to half-pints, and the number of these be divided by 7, the quotient must be the number required; thus:

|                                       |
|---------------------------------------|
| 126 <i>gals.</i>                      |
| <u>4</u>                              |
| 504 <i>quarts.</i>                    |
| <u>4</u>                              |
| 7)2016 <i>half-pints.</i>             |
| <u>288</u> <i>the number of each.</i> |

| <i>oz.</i> <i>dwt.</i>             |
|------------------------------------|
| (16) 2 4                           |
| <u>20</u>                          |
| 44 <i>dwt. in each.</i>            |
| <u>24</u>                          |
| 176                                |
| <u>88</u>                          |
| 1056 <i>grs. in each.</i>          |
| <u>36</u>                          |
| 6336                               |
| <u>3168</u>                        |
| 38016 <i>grs. in 36, or 3 doz.</i> |

$$\begin{array}{r}
 \text{ac. rood. per.} \\
 (17) \quad 2 \quad 0 \quad 16 \\
 \quad \quad 4 \\
 \quad \quad \hline
 \quad \quad 8 \\
 \quad \quad 40 \\
 \quad \quad \hline
 4)336 \\
 \quad 30\frac{1}{2} \\
 \quad \hline
 10080 \\
 \quad 84 \\
 \quad \hline
 10164 \text{ square yards.} \\
 \quad 9 \\
 \quad \hline
 91476 \text{ square feet.}
 \end{array}$$

$$\begin{array}{r}
 \text{bar.} \\
 (18) \quad 12000 \\
 \quad 36^* \\
 \quad \hline
 432000 \text{ gals.} \\
 \quad 4 \\
 \quad \hline
 1728000 \text{ qts.}
 \end{array}$$

$$\begin{array}{r}
 \text{£.} \\
 (19) \quad 505107 \\
 \quad \quad 20 \\
 \quad \hline
 8,0)1010214,0 \text{ num. of shillings.} \\
 6,0)12627,7 \text{ (nearly) minutes.} \\
 12)2104 \text{ hours.} \\
 \quad 175 \text{ days.} \\
 \text{Jan. 31 days.} \\
 \text{Feb. 29} \\
 \text{March 31} \\
 \text{April 30} \\
 \text{May 31}
 \end{array}$$

152 days from Jan. 1 to the end of May.

175—152=23, therefore 23 days are occupied after the end of May, so that the labour is not completed till the 23d of June. The counting must be continued for 4 hours 37 minutes after the expiration of the 12 hours on that day, since 4 hours 37 minutes have been disregarded in the above work.

$$\begin{array}{r}
 \text{lbs. troy.} \\
 (20) \quad 1751 \\
 \quad 12 \\
 \quad \hline
 21012 \\
 \quad 20 \\
 \quad \hline
 420240 \\
 \quad 24 \\
 \quad \hline
 1680960 \\
 84048 \\
 \hline
 123)10085760(81998 \text{ sows.} \\
 \quad 984 \\
 \quad \hline
 \quad 245 \\
 \quad 123 \\
 \quad \hline
 \quad 1227 \\
 \quad 1107 \\
 \quad \hline
 \quad 1206 \\
 \quad 1107 \\
 \quad \hline
 \quad 990 \\
 \quad 984 \\
 \quad \hline
 \quad 6
 \end{array}$$

\* Here it is convenient to consider the upper number, 12, to be the multiplier, and the lower, 36, to be the multiplicand: that is, to take 12 times 36 rather than 36 times 12; and in all cases where two numbers are placed one below the other for multiplication, that which appears the more convenient should always be taken for multiplier, whether it be the lower or the upper number.

*Reduction of Quantities to Higher Denominations,*  
pages 51, 52.

(1) 4)26493 *far.*

12) 66231

20)55,1 11d.

£27 11s. 11½d.

(2) 397024 yds.

2\*

11) 794048 *half yds.*

4,0)7218,6...1 yd. \*

8) 1804 ... 26 per.

225 m. 4 fur. 26 per. 1 yd.

(3) 6,0)2863,5 sec.

6,0)47,7...15

7h. 57 m. 15 sec.

(4) 6) 12875 grs.

$$24 = 6 \times 4$$

4) 2145...5 grs.

$2.0)53.6 \div \dots 6 \text{ grs.}$

12) 26...16 dwt.

2 lb. 2 oz. 16 dwt. 11 grs.

(5) 8)176432 lbs.

$$112 = 8 \times 7 \times 2$$

7)22054

2) 3150...32 lbs. = 1 qr. 4 lbs.

2,0)157,5

78 tons, 15 cwt. 1 qr. 4 lbs.

(6) 4)24631 *nails*.

4) 6157...3 na.

1539 yds. 1 qr. 3 na.

(7) 4,0)4265,7

4)1066...17 po.

266 ac. 2 roo. 17 po.

\* See the Arithmetic, page 50.

† The remainder here is 1; but this is not 1 grain: it is once six grains. See pages 49, 50, Arithmetic.

(8)  $12 \overline{)27568} \text{ sq. in.}$

$12 \overline{)2297} \dots 4 \text{ in.}$

$9 \overline{)191} \dots 60 \text{ in.}$

$\underline{21} \text{ sq. yds. } 2 \text{ ft. } 64 \text{ in.}$

(9) Since 8 pints make 1 gallon,  
the work is as follows :—

$8 \overline{)100000} \text{ pints.}$

$\underline{12500} \text{ gals.}$

(10)  $6,0 \overline{)13253,0''}$

$6,0 \overline{)220,8} \dots 50''$

$\underline{36^\circ} \text{ } 48' \text{ } 50''$

(11)  $12 \overline{)100000} \text{ c. in.}$

$12 \overline{)8333} \dots 4 \text{ in.}$

$12 \overline{)694} \dots 60 \text{ in.}$

$3 \overline{)57} \dots 1440 \text{ in.}$

$9 \overline{)19}$

$\underline{2} \text{ c. yds. } 3 \text{ ft. } 1504 \text{ in.}$

In Example 11, the three divisors, 12, are used instead of 1728, the number of cubic inches in a cubic foot ; because  $1728 = 12 \times 12 \times 12$ . The first division by 12 gives for quotient 8333, so that each unit in this number is 12 inches ; the next division gives for quotient 694, and for remainder 5, that is to say, five of the units just spoken of—units of 12 inches in value each—so that this remainder, 5, is equal to  $5 \times 12 = 60$  inches ; and the units in the quotient, 694, are each 144 inches in value. The next division by 12 gives 57 for quotient, and 10 of the last-mentioned units for remainder ; so that the remainder in inches is  $10 \times 144 = 1440$ , the denomination of the quotient, 57, being cubic feet, which are divided by 27, or  $3 \times 9$ , to get cubic yards ; the result is 2 cubic yards, 3 feet, and  $1440 + 60 + 4$  inches.

(12)  $8 \overline{)100000} \text{ lbs.}$

$112 = 8 \times 7 \times 2 \quad 7 \overline{)12500}$

$2 \overline{)1785} \dots 40 \text{ lbs.}$

$2,0 \overline{)89,2} \dots 56 \text{ lbs.}$

$\underline{44} \text{ tons } 12 \text{ cwt. } 3 \text{ qrs. } 12 \text{ lbs.}$

Otherwise :

$4 \overline{)100000} \text{ lbs.}$

$28 = 4 \times 7 \quad 7 \overline{)25000}$

$4 \overline{)3571} \dots 12 \text{ lbs.}$

$2,0 \overline{)89,2} \dots 3 \text{ qrs.}$

$\underline{44} \text{ tons } 12 \text{ cwt. } 3 \text{ qrs. } 12 \text{ lbs.}$

$$\begin{array}{r}
 (13) \quad 1000 \\
 \quad \underline{52} \text{ weeks in a year.} \\
 4) \underline{52000} \\
 2,0) \underline{1300,0} \text{ shillings.} \\
 \quad \underline{\pounds 650}
 \end{array}$$

$$\begin{array}{r}
 (14) \quad 12) \underline{7239962} \text{ pence.} \\
 \quad 2,0) \underline{60333,0} \dots 2d. \\
 \quad \quad \underline{\pounds 30166} \text{ 10s. 2d.}
 \end{array}$$

$$\begin{array}{r}
 (15) \\
 \quad \underline{3000} \\
 \quad \underline{2} \\
 19) \underline{6000} (315 \frac{15}{19} \text{ hours.} \\
 \quad \underline{57} \quad \text{ho.} \\
 \quad \quad 30 \quad 24) \underline{315} (13 \text{ days.} \\
 \quad \quad \underline{19} \quad \quad 24 \\
 \quad \quad \underline{110} \quad \quad 75 \\
 \quad \quad \underline{95} \quad \quad 72 \\
 \quad \quad \underline{15} \quad \quad 3 \text{ hours.}
 \end{array}$$

Therefore the time is 13 *days*  
 $3 \frac{15}{19}$  *hours*.

$$\begin{array}{r}
 (16) \\
 \quad \underline{9080} \\
 \quad \underline{2} \\
 19) \underline{18160} (955 \frac{15}{19} \text{ hours.} \\
 \quad \underline{171} \quad \text{ho.} \\
 \quad \quad 106 \quad 24) \underline{955} (39 \text{ days.} \\
 \quad \quad \underline{95} \quad \quad 72 \\
 \quad \quad \underline{110} \quad \quad 235 \\
 \quad \quad \underline{95} \quad \quad 216 \\
 \quad \quad \underline{15} \quad \quad 19 \text{ hours.}
 \end{array}$$

Therefore the time is 39 *days*  
 $19 \frac{15}{19}$  *hours*.

In each of the last two Examples we have to divide by  $9 \frac{1}{2}$  to get the number of hours, because the number of hours must be the same as the number of times the proposed distance contains  $9 \frac{1}{2}$  miles. For the purpose of simplifying the work, *twice* the distance in each case is divided by *twice*  $9 \frac{1}{2}$ , or 19. (Page 50, Arithmetic.)

$$\begin{array}{r}
 (17) \quad 8) \underline{448081} \text{ lbs.} \\
 112 = 8 \times 2 \times 7 \quad 2) \underline{56010} \dots 1 \text{ lb.} \\
 \quad \quad \quad 7) \underline{28005} \\
 \quad \quad 2,0) \underline{400,0} \dots 80 \text{ lbs.} \\
 \quad \quad \quad \underline{200 \text{ t. 0 cwt. 81 lbs.}}
 \end{array}$$

Otherwise :

$$\begin{array}{r}
 112 \\
 \underline{20} \\
 224,0) \underline{44808,1} (200 \text{ tons, 81 lbs.} \\
 \quad \underline{448} \\
 \quad \quad \underline{81 \text{ lbs.}}
 \end{array}$$

$$\begin{array}{r}
 (18) \quad 891650 \\
 \underline{875631} \\
 2)1767281^* s. \\
 \underline{883640\frac{1}{2} s.} \\
 2,0)265092,1...6d. \\
 \underline{\pounds 132546} \quad 1s. \quad 6d.
 \end{array}$$

|                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                   |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $  \begin{array}{r}  (19) \quad 2 \text{ ft. } 8 \text{ in.} \\  \underline{12} \\  32 \\  \underline{108} \\  256 \\  \underline{32} \\  3456 \text{ inches marched} \\  \text{in one minute.}  \end{array}  $ | $  \begin{array}{r}  10 \text{ miles} = 80 \text{ furlongs.} \\  \underline{40} \\  2)3200 \text{ poles.} \\  \underline{5\frac{1}{2}} \\  16000 \\  \underline{1600} \\  17600 \text{ yds.} \\  \underline{3} \\  52800 \text{ feet.} \\  \underline{12} \\  633600 \text{ inches in ten miles.}  \end{array}  $ |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

$$\begin{array}{r}
 3456)633600(183 \text{ min. } 20 \text{ sec., or,} \\
 \underline{3456} \quad \text{dividing 183 by 60,} \\
 28800 \quad 3 \text{ ho. } 3 \text{ min. } 20 \text{ sec.} \\
 \underline{27648} \\
 11520 \\
 \underline{10368} \\
 1152 \\
 \underline{60} \\
 69120 \\
 \underline{69120}
 \end{array}$$

It is plain that as many times as the distance marched in one minute is contained in the whole distance, so many minutes must be spent in marching that distance; the number of minutes is therefore  $183\frac{1152}{3456}$ ; that is, it is 183 minutes and 1152 minutes divided by 3456, or  $1152 \times 60$  seconds divided by 3456, which is 20 seconds.

(20) From Example 18, p. 47, it appears that there are 432000 gallons, old measure, in the vat; these are brought to imperial gallons by multiplying them by 60, and then dividing by 59. The

\* This is the total number of advertisements; consequently the duty is so many *skillings* and half as many more.



number of imperial gallons multiplied by 10 will give the number of pounds weight; therefore the work is as follows:—

|                             |                             |                                      |
|-----------------------------|-----------------------------|--------------------------------------|
| 432000                      |                             |                                      |
| <u>60</u> <i>Imp. gals.</i> |                             | <i>lbs.</i>                          |
| 59)25920000(439322          |                             | 2)4393220                            |
| 236                         | $112 = 2 \times 8 \times 7$ | 8)2196610                            |
| 232                         |                             | 7)274576... 4 <i>lbs.</i>            |
| <u>177</u>                  |                             | 2,0)39225...16 <i>lbs.</i>           |
| 550                         |                             | <u>1961</u> <i>t. 5 cwt. 20 lbs.</i> |
| <u>531</u>                  |                             |                                      |
| 190                         |                             |                                      |
| <u>177</u>                  |                             |                                      |
| 130                         |                             |                                      |
| <u>118</u>                  |                             |                                      |
| 120                         |                             |                                      |
| <u>118</u>                  |                             |                                      |
| <u>2</u>                    |                             |                                      |

(21) It appears, from the work of last Example, that the vessel will contain 4393220 lbs. of water; we shall therefore have to reduce these to ounces, and to divide by 1000 oz. to get the number of cubic feet:—

|                            |  |                                                                    |
|----------------------------|--|--------------------------------------------------------------------|
| 4393220 <i>lbs.</i>        |  | 1,000)70291,520                                                    |
| <u>16</u>                  |  | 9)70291 <sup>280</sup> / <sub>1000</sub> <i>cubic feet.</i>        |
| 26359320                   |  | 3)7810...1 <i>c. ft.</i>                                           |
| <u>439322</u>              |  | 2603 <i>c. yds.</i> 10 <sup>320</sup> / <sub>1000</sub> <i>ft.</i> |
| <u>70291520</u> <i>oz.</i> |  |                                                                    |

that is 260 *c. yds.* 10 $\frac{1}{2}$  *ft.* \*

(22) As the vessel contains 439322 imperial gallons, it contains double that number of half-gallons; and by the question the number of half-gallons is also the number of seconds occupied in discharging them. So that, if we multiply 439322 by 2, and divide the product by 60, we shall have the number of minutes; but this is the same as to divide only by 30, and not multiply at all; therefore,

|                                                           |
|-----------------------------------------------------------|
| 3,0)43932,2 <i>units of two seconds each.</i>             |
| 6,0)1464,4...4 <i>sec.</i>                                |
| 24)244...4 <i>m.</i>                                      |
| <u>10</u> <i>d.</i> 4 <i>h.</i> 4 <i>m.</i> 4 <i>sec.</i> |

\* It is plain that if the remainder 520 be multiplied by 2, and then divided by the 1000, the result will be the number of half-feet besides

| (23)            | lbs.        | (24)           |                     |
|-----------------|-------------|----------------|---------------------|
|                 | 8)949760    | 176,0*)64240\0 | (365 miles, the pay |
| 112 = 8 × 2 × 7 | 2)118720    | 528            | for which is the    |
|                 | 7)59360     | 1144           | same as for 400     |
|                 | 2)8480 cwt. | 1056           | miles: that is,     |
|                 | 4240 sacks. | 880            | 4 times 2s. 6d.,    |
|                 |             | 880            | or 10s.             |

~~~~~

### *Addition of Compound Quantities, pages 54, 55.*

The only assistance that a Key can afford to a learner in working these Examples is to furnish him with the *carryings* from the several columns; these only are therefore put down, each set of carryings against the number of the Example.

Carryings.		Carryings.	
(1) £2 2s. 2d.		(11) 3 oz. 3 dwt.	
(2) £3 3s. 3d.		(12) 2 lbs. 3 oz. 2 dwt.	
(3) £4 3s. 2d.		(13) 3 dr. 3 scr.	
(4) 3 d. 2 h.		(14) 2 oz. 1 dr.	
(5) 3 d. 2 h.		(15) 4 lbs. 3 oz. 2 dr. 2 scr.	
(6) 4 d. 1 h. 2 m.		(16) 2 yds. 3 ft.	
(7) 3 oz. 3 dr.		(17) 2 fur. 3 po. 2 yds.	
(8) 2 qrs. 2 lbs.		(18) 2 m. 1 fur. 2 po.	
(9) 2 cwt. 2 qrs. 3 lbs.		(19) 3 ac. 3 roo. 4 per.	
(10) 3 oz. 3 dwt.		(20) 2 ac. 3 roo. 3 per.	

~~~~~

### *Subtraction of Compound Quantities, pages 56, 57, 58.*

As in Addition, the *carryings* only are inserted, each set being placed against the number of the Example:—

| Carryings.         |  | Carryings.             |  |
|--------------------|--|------------------------|--|
| (1) £1 1s. 0d.     |  | (7) 0 yd. 0 ft.        |  |
| (2) £1 1s. 1d.     |  | (8) 1 yd. 1 ft.        |  |
| (3) £1 1s. 0d.     |  | (9) 0 per. 0 yd. 1 ft. |  |
| (4) 1 d. 1 h.      |  | (10) 1° 1'             |  |
| (5) 1 d. 1 h.      |  | (11) 1° 1'             |  |
| (6) 1 d. 1 h. 1 m. |  | (12) 1° 1'             |  |

whole feet in the complete quotient; so that there is only 1 half-foot, and a small remainder, not worth notice, over.

\* See Example 1, page 4, of the Arithmetic.

| <i>Carryings.</i> |                                                       | <i>Carryings.</i> |                                                    |
|-------------------|-------------------------------------------------------|-------------------|----------------------------------------------------|
| (13)              | 1 d. 1 h. 1 m.                                        | (26)              | 1 yd. 1 ft. <del>8</del> 1 c. ft.<br>= 1728 c. in. |
| (14)              | 1 t. 0 cwt. 1 qr.                                     | (27)              | 1 yd. 1 ft.                                        |
| (15)              | 0 t. 1 cwt. 1 qr.                                     | (28)              | 1 yd. 1 ft.                                        |
| (16)              | 1 t. 0 cwt. 1 qr.                                     | (29)              | 1 yd. 1 ft.                                        |
| (17)              | 0 m. 1 fur. 1 per.                                    | (30)              | 1 yd. 1 ft.                                        |
| (18)              | 1 m. 1 fur. 1 per.                                    | (31)              | 1 yd. 1 ft.                                        |
| (19)              | 1 m. 1 fur. 1 per.                                    | (32)              | 0 gal. 1 qt.                                       |
| (20)              | 1 ac. 1 roo. 1 per.                                   | (33)              | 1 gal. 0 qt.                                       |
| (21)              | 1 ac. 1 roo. 2 per. <del>8</del> 2 per.<br>= 60½ yds. | (34)              | 1 gal. 1 qt.                                       |
| (22)              | 1 ac. 1 roo. 1 per.                                   | (35)              | 0 pk. 1 gal. 0 qt.                                 |
| (23)              | 1 oz. 1 dwt.                                          | (36)              | 1 bush. 1 pk. 1 gal.                               |
| (24)              | 1 lb. 1 oz. 1 dwt.                                    | (37)              | 1 bush. 1 pk. 1 gal.                               |
| (25)              | 1 lb. 1 oz. 0 dwt.                                    |                   |                                                    |

*Multiplication of Compound Quantities, pages 60, 61.*

| £     | s. | d. | £     | s.  | d. | £     | s. | d.  |     |    |    |
|-------|----|----|-------|-----|----|-------|----|-----|-----|----|----|
| (1)   | 32 | 8  | 6½    | (2) | 43 | 11    | 4½ | (3) | 125 | 13 | 0½ |
|       |    | 5  |       |     |    | 8     |    |     |     | 12 |    |
| <hr/> |    |    | <hr/> |     |    | <hr/> |    |     |     |    |    |
| £164  | 2  | 8½ | £348  | 11  | 2  | £1507 | 16 | 3   |     |    |    |

|     | £     | s. | d. |     | £     | s. | d. |
|-----|-------|----|----|-----|-------|----|----|
| (4) | 217   | 18 | 9½ | (5) | 734   | 19 | 7½ |
|     |       |    | 11 |     |       |    | 9  |
|     | £2397 | 6  | 5½ |     | £6614 | 16 | 9½ |

(6)  $28 = 4 \times 7$

| £    | s. | d. |
|------|----|----|
| 19   | 13 | 5¼ |
|      |    | 4  |
| 78   | 13 | 9  |
|      |    | 7  |
| £550 | 16 | 3  |

Otherwise :

| £   | s. | d. |       | £    | s. | d. |
|-----|----|----|-------|------|----|----|
| 19  | 13 | 5¼ | × 8 = | 157  | 7  | 6  |
|     |    | 10 |       |      |    |    |
| 196 | 14 | 4½ | × 2 = | 393  | 8  | 9  |
|     |    |    |       |      |    |    |
|     |    |    |       | £550 | 16 | 3  |

(7)  $343 = 7 \times 7 \times 7$

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 21 \quad 9 \quad 10\frac{1}{2} \\
 \hline
 7 \\
 150 \quad 9 \quad 1\frac{1}{2} \\
 \hline
 7 \\
 1053 \quad 3 \quad 10\frac{1}{2} \\
 \hline
 7 \\
 \text{£}7372 \quad 7 \quad 1\frac{1}{2}
 \end{array}$$

Otherwise :

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 21 \quad 9 \quad 10\frac{1}{2} \times 3 = 64 \quad 9 \quad 7\frac{1}{2} \\
 \hline
 10 \\
 214 \quad 18 \quad 9 \times 4 = 859 \quad 15 \quad 0 \\
 \hline
 10 \\
 2149 \quad 7 \quad 6 \times 3 = 6448 \quad 2 \quad 6 \\
 \hline
 \text{£}7372 \quad 7 \quad 1\frac{1}{2}
 \end{array}$$

(8)  $504 = 9 \times 8 \times 7$

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 32 \quad 17 \quad 1\frac{3}{4} \\
 \hline
 9 \\
 295 \quad 14 \quad 3\frac{3}{4} \\
 \hline
 8 \\
 2365 \quad 14 \quad 6 \\
 \hline
 7 \\
 \text{£}16560 \quad 1 \quad 6
 \end{array}$$

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 32 \quad 17 \quad 1\frac{3}{4} \times 4 = 131 \quad 8 \quad 7 \\
 \hline
 10 \\
 328 \quad 11 \quad 5\frac{1}{2} \\
 \hline
 10 \\
 3285 \quad 14 \quad 7 \times 5 = 16428 \quad 12 \quad 11 \\
 \hline
 \text{£}16560 \quad 1 \quad 6
 \end{array}$$

(9)  $891 = 11 \times 9 \times 9$

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 103 \quad 11 \quad 8\frac{1}{4} \\
 \hline
 11 \\
 1139 \quad 8 \quad 6\frac{3}{4} \\
 \hline
 9 \\
 10254 \quad 17 \quad 0\frac{3}{4} \\
 \hline
 9 \\
 \text{£}92293 \quad 13 \quad 6\frac{3}{4}
 \end{array}$$

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 103 \quad 11 \quad 8\frac{1}{4} \times 1 = 103 \quad 11 \quad 8\frac{1}{4} \\
 \hline
 10 \\
 1035 \quad 16 \quad 10\frac{1}{2} \times 9 = 9322 \quad 11 \quad 10\frac{1}{2} \\
 \hline
 10 \\
 10358 \quad 8 \quad 9 \times 8 = 82867 \quad 10 \quad 0 \\
 \hline
 \text{£}92293 \quad 13 \quad 6\frac{3}{4}
 \end{array}$$

(10)  $1617 = 11 \times 7 \times 7 \times 3$

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 379 \quad 18 \quad 7\frac{3}{4} \\
 \hline
 11 \\
 4179 \quad 5 \quad 1\frac{1}{4} \\
 \hline
 7 \\
 29254 \quad 15 \quad 8\frac{3}{4} \\
 \hline
 7 \\
 204783 \quad 10 \quad 1\frac{1}{4} \\
 \hline
 3 \\
 \text{£}614350 \quad 10 \quad 3\frac{3}{4}
 \end{array}$$

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 379 \quad 18 \quad 7\frac{3}{4} \times 7 = 2659 \quad 10 \quad 6\frac{1}{4} \\
 \hline
 10 \\
 3799 \quad 6 \quad 5\frac{1}{2} \times 1 = 3799 \quad 6 \quad 5\frac{1}{2} \\
 \hline
 10 \\
 37993 \quad 4 \quad 7 \times 6 = 227959 \quad 7 \quad 6 \\
 \hline
 10 \\
 379932 \quad 5 \quad 10 \times 1 = 379932 \quad 5 \quad 10 \\
 \hline
 \text{£}614350 \quad 10 \quad 3\frac{3}{4}
 \end{array}$$

(11)  $75 = 5 \times 5 \times 3$

Otherwise :

| <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>       |  | <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>     |              | <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>     |
|------------|-------------|-------------|-------------------|--|------------|-------------|-------------|-----------------|--------------|------------|-------------|-------------|-----------------|
| 15         | 3           | 2           | 4*                |  | 15         | 3           | 2           | 4               | $\times 5 =$ | 76         | 7           | 13          | 3 $\frac{1}{2}$ |
|            |             |             | 5                 |  |            |             |             | 10              |              |            |             |             |                 |
| 76         | 7           | 13          | 3 $\frac{1}{2}$ * |  | 153        | 6           | 27          | 1 $\frac{1}{2}$ | $\times 7 =$ | 1076       | 6           | 30          | 5               |
|            |             |             | 5                 |  |            |             |             |                 |              |            |             |             |                 |
| 384        | 4           | 28          | 1                 |  |            |             |             |                 |              | 1153       | 6           | 4           | 3               |
|            |             |             | 3                 |  |            |             |             |                 |              |            |             |             |                 |
| 1153       | 6           | 4           | 3                 |  |            |             |             |                 |              |            |             |             |                 |

(12)  $98 = 7 \times 7 \times 2$

| <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>     |                                         | <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>     |              | <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>     |
|------------|-------------|-------------|-----------------|-----------------------------------------|------------|-------------|-------------|-----------------|--------------|------------|-------------|-------------|-----------------|
| 2)17       | 7           | 0           | 5               |                                         | 2)17       | 7           | 0           | 5               | $\times 8 =$ | 143        | 0           | 7           | 1 $\frac{1}{2}$ |
|            |             |             | 7               |                                         |            |             |             | 10              |              |            |             |             |                 |
| 125        | 1           | 6           | 2               |                                         | 178        | 6           | 9           | 0 $\frac{1}{2}$ | $\times 9 =$ | 1609       | 0           | 1           | 4 $\frac{1}{2}$ |
|            |             |             | 7               |                                         |            |             |             |                 |              |            |             |             |                 |
| 876        | 0           | 4           | 3               |                                         |            |             |             |                 |              |            |             |             |                 |
|            |             |             | 2               |                                         |            |             |             |                 |              |            |             |             |                 |
| 1752       | 0           | 9           | 0 $\frac{1}{2}$ |                                         |            |             |             |                 |              |            |             |             |                 |
| 8          | 7           | 20          | 2 $\frac{1}{2}$ | <i>for the <math>\frac{1}{2}</math></i> |            |             |             |                 |              |            |             |             |                 |
| 1760       | 7           | 29          | 3               |                                         |            |             |             |                 |              |            |             |             |                 |

(13)  $256 = 8 \times 8 \times 4$

| <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>     |  | <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>     |              | <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i> |
|------------|-------------|-------------|-----------------|--|------------|-------------|-------------|-----------------|--------------|------------|-------------|-------------|-------------|
| 23         | 1           | 31          | 2               |  | 23         | 1           | 31          | 2               | $\times 6 =$ | 139        | 2           | 28          | 1           |
|            |             |             | 8               |  |            |             |             | 10              |              |            |             |             |             |
| 185        | 6           | 10          | 6               |  | 232        | 1           | 33          | 3 $\frac{1}{2}$ | $\times 5 =$ | 1161       | 1           | 8           | 1           |
|            |             |             | 8               |  |            |             |             | 10              |              |            |             |             |             |
| 1486       | 2           | 7           | 1 $\frac{1}{2}$ |  | 2322       | 2           | 16          | 2               | $\times 2 =$ | 4644       | 4           | 32          | 4           |
|            |             |             | 4               |  |            |             |             |                 |              |            |             |             |             |
| 5945       | 0           | 29          | 0 $\frac{1}{2}$ |  |            |             |             |                 |              |            |             |             |             |

\* 4 yd.  $\times 5 = 20$  yd.  $= 40$  half-yards, therefore dividing 40 by 11, because 11 half-yards make 1 perch, we get for quotient 3 and 7 over; that is to say, the 40 half-yards are equal to 3 perches and 7 half-yards, or 3 per. 3 $\frac{1}{2}$  yd. In like manner 3 $\frac{1}{2}$  yd.  $\times 5 = 17\frac{1}{2}$  yd., and  $17\frac{1}{2} \div 5\frac{1}{2}$  is the same as  $35 \div 11$ , namely, 3 and 2 over; that is, 2 halves over, or 1 whole; hence, in multiplying the yards in operations like that above, the learner should consider the multiplicand as *doubled*, and

(14)  $594 = 11 \times 9 \times 6$

Otherwise :

| <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i> | <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>               |              | <i>mi.</i> | <i>fur.</i> | <i>per.</i>    | <i>yds.</i> |
|------------|-------------|-------------|-------------|------------|-------------|-------------|---------------------------|--------------|------------|-------------|----------------|-------------|
| 0          | 6           | 27          | 3           | 0          | 6           | 27          | 3                         | $\times 4 =$ | 3          | 2           | 30             | 1           |
|            |             |             | 11          |            |             |             | 10                        |              |            |             |                |             |
| 9          | 1           | 23          | 0           | 8          | 2           | 35          | $2\frac{1}{2} \times 9 =$ | 75           | 1          | 39          | $0\frac{1}{2}$ |             |
|            |             |             | 9           |            |             |             | 10                        |              |            |             |                |             |
| 82         | 6           | 7           | 0           | 83         | 4           | 34          | $3 \times 5 =$            | 418          | 0          | 12          | 4              |             |
|            |             |             | 6           |            |             |             |                           |              |            |             |                |             |
| 496        | 5           | 2           | 0           |            |             |             |                           |              | 496        | 5           | 2              | 0           |

(15)  $605 = 11 \times 11 \times 5$

| <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i> | <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>    |              | <i>mi.</i> | <i>fur.</i> | <i>per.</i>    | <i>yds.</i> |
|------------|-------------|-------------|-------------|------------|-------------|-------------|----------------|--------------|------------|-------------|----------------|-------------|
| 1          | 5           | 19          | 1           | 1          | 5           | 19          | 1              | $\times 5 =$ | 8          | 3           | 15             | 5           |
|            |             |             | 11          |            |             |             | 10             |              |            |             |                |             |
| 18         | 4           | 11          | 0           | 16         | 6           | 31          | $4\frac{1}{2}$ |              |            |             |                |             |
|            |             |             | 11          |            |             |             | 10             |              |            |             |                |             |
| 203        | 7           | 1           | 0           | 168        | 3           | 38          | $1 \times 6 =$ | 1010         | 7          | 29          | $0\frac{1}{2}$ |             |
|            |             |             | 5           |            |             |             |                |              |            |             |                |             |
| 1019       | 3           | 5           | 0           |            |             |             |                |              | 1019       | 3           | 5              | 0           |

(16)  $972 = 12 \times 9 \times 9$

| <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i> | <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>               |              | <i>mi.</i> | <i>fur.</i> | <i>per.</i> | <i>yds.</i>    |
|------------|-------------|-------------|-------------|------------|-------------|-------------|---------------------------|--------------|------------|-------------|-------------|----------------|
| 27         | 3           | 22          | 4           | 27         | 3           | 22          | 4                         | $\times 2 =$ | 54         | 7           | 5           | $2\frac{1}{2}$ |
|            |             |             | 12          |            |             |             | 10                        |              |            |             |             |                |
| 320        | 2           | 32          | 4           | 274        | 3           | 27          | $1\frac{1}{2} \times 7 =$ | 1921         | 1          | 30          | 5           |                |
|            |             |             | 9           |            |             |             | 10                        |              |            |             |             |                |
| 2964       | 1           | 14          | 3           | 2744       | 4           | 32          | $4 \times 9 =$            | 24701        | 3          | 14          | 3           |                |
|            |             |             | 9           |            |             |             |                           |              |            |             |             |                |
| 26677      | 4           | 10          | 5           |            |             |             |                           |              | 26677      | 4           | 10          | 5              |

then dividing the product by 11, he will have for quotient the proper number to be carried to the perches, and for remainder the number of half-yards over. Thus, in the second line of the work above, he should say, not 5 times  $3\frac{1}{2}$ , but 5 times 7, the double of  $3\frac{1}{2}$ . In like manner in Example 12, the first step is 7 times 10, which gives 70, and this, divided by 11, gives 6 to be carried to the perches, and 4 over, which is the number of half-yards.

(17)  $441 = 9 \times 7 \times 7$

Otherwise :

| <i>da. ho. min.</i>                             | <i>da. ho. min.</i>                 | <i>da. ho. min.</i> |
|-------------------------------------------------|-------------------------------------|---------------------|
| 4)19 13 27<br>9                                 | 4)19 13 27 $\times 1 =$             | 19 13 27            |
| 176 1 3<br>7                                    | 195 14 30 $\times 4 =$              | 782 10 0            |
| 1232 7 21<br>7                                  | 1956 1 0 $\times 4 =$               | 7824 4 0            |
| 8626 3 27                                       | <i>for</i> $\frac{1}{4}$ 4 21 21 45 |                     |
| <i>for</i> $\frac{1}{4}$ 4 21 21 45 <i>sec.</i> | 8631 0 48 45                        |                     |
| 8631 0 48 45                                    | 8631 0 48 45                        |                     |

(18)  $231 = 11 \times 7 \times 3$

|                                                         |                                      |
|---------------------------------------------------------|--------------------------------------|
| 4)16° 51' 43" ( 4° 12' 55" 45" <i>for</i> $\frac{1}{4}$ |                                      |
| 11                                                      | 3                                    |
| 185 28 53                                               | 12 38 47 15 <i>for</i> $\frac{3}{4}$ |
| 7                                                       |                                      |
| 1298 22 11                                              |                                      |
| 3                                                       |                                      |
| 3895 6 33                                               |                                      |
| <i>add for</i> $\frac{3}{4}$ 12 38 47 $\frac{1}{4}$     |                                      |
| 3907 45 20 $\frac{1}{4}$                                |                                      |

(19)  $243 = 9 \times 9 \times 3$

| <i>t. cwt. grs. lbs.</i> | <i>t. cwt. grs. lbs.</i> | <i>t. cwt. grs. lbs.</i> |
|--------------------------|--------------------------|--------------------------|
| 14 13 2 11<br>9          | 14 13 2 11 $\times 3 =$  | 44 0 3 5                 |
| 132 2 1 15<br>9          | 146 15 3 26 $\times 4 =$ | 587 3 3 20               |
| 1189 1 1 23<br>3         | 1467 19 3 8 $\times 2 =$ | 2935 19 2 16             |
| 3567 4 1 13              |                          | 3567 4 1 13              |

(20)  $511 = 8 \times 8 \times 8 - 1$

Otherwise :

| <i>ac. roo. po.</i>                                | $\times 1$ | <i>ac. roo. po.</i>   | $\times 1 =$ | <i>ac. roo. po.</i> |
|----------------------------------------------------|------------|-----------------------|--------------|---------------------|
| <u>13 3 17</u><br>8                                |            | <u>13 3 17</u><br>10  |              | <u>13 3 17</u>      |
| <u>110 3 16</u><br>8                               |            | <u>138 2 10</u><br>10 |              | <u>138 2 10</u>     |
| <u>886 3 8</u><br>8                                |            | <u>1385 2 20</u>      | $\times 5 =$ | <u>6928 0 20</u>    |
| <i>subtract</i> <u>7094 1 24</u><br><u>13 3 17</u> |            |                       |              | <u>7080 2 7</u>     |
| <u>7080 2 7</u>                                    |            |                       |              |                     |

(21)  $563 = 9 \times 9 \times 7 - 4$

| <i>sq. yd. ft. in.</i>                             | $\times 4$ | <i>sq. yd. ft. in.</i> | $\times 3 =$ | <i>sq. yd. ft. in.</i> |
|----------------------------------------------------|------------|------------------------|--------------|------------------------|
| <u>2 8 123</u><br>9                                |            | <u>2 8 123</u><br>10   |              | <u>8 8 81</u>          |
| <u>26 7 99</u><br>9                                |            | <u>29 7 78</u><br>10   | $\times 6 =$ | <u>179 0 36</u>        |
| <u>241 6 27</u><br>7                               |            | <u>298 3 60</u>        | $\times 5 =$ | <u>1491 8 12</u>       |
| <i>subtract</i> <u>1691 7 45</u><br><u>11 8 60</u> |            |                        |              | <u>1679 7 129</u>      |
| <u>1679 7 129</u>                                  |            |                        |              |                        |

(22)  $616 = 11 \times 8 \times 7$

| <i>oz. dwt. grs.</i>                                      | $\times 6 =$             | <i>oz. dwt. grs.</i>                   |
|-----------------------------------------------------------|--------------------------|----------------------------------------|
| <u>4 ) 9 17 20</u><br>11                                  | <u>4 ) 9 17 20</u><br>10 | <u>59 7 0</u>                          |
| <u>108 16 4</u><br>8                                      | <u>98 18 8</u><br>10     | $\times 1 =$ <u>98 18 8</u>            |
| <u>870 9 8</u><br>7                                       | <u>989 3 8</u>           | $\times 6 =$ <u>5935 0 0</u>           |
| <i>for</i> $\frac{1}{4}$ <u>6093 5 8</u><br><u>2 9 11</u> |                          | <i>for</i> $\frac{1}{4}$ <u>2 9 11</u> |
| <u>6095 14 19</u>                                         |                          | <u>6095 14 19</u>                      |



*Division of Compound Quantities, pages 64, 65, 66.*

| (1)        |           |           | (2)   |    |           |     |       |    |           |
|------------|-----------|-----------|-------|----|-----------|-----|-------|----|-----------|
| £          | s.        | d.        | £     | s. | d.        | £   | s.    | d. |           |
| 8)148      | 16        | 4         | 7)237 | 13 | 5         | or, | 2)237 | 13 | 5         |
| <u>£18</u> | <u>12</u> | <u>0½</u> | 2)33  | 19 | 0½        |     | 7)118 | 16 | 8½        |
|            |           |           | £16   | 19 | 6¼ + ¾ f. |     | £16   | 19 | 6¼ + ¾ f. |

In the second example the factors of 14 are employed instead of 14 itself; and by changing the order in which these factors are applied a different form is given to the work. This difference of form is here exhibited, to show the learner that it is often more convenient to take the factors in one order rather than in another. In the first mode of working, the division by 7 leads to the fraction  $\frac{5}{7}$ , that is, 5 pence divided by 7, which is 20 farthings divided by 7; the half of this, given by the next division by 2, is 10 farthings divided by 7, that is, 1 farthing and  $\frac{3}{7}$ , the  $\frac{3}{7}$  being neglected in the answer, as fractions of a farthing are disregarded in the final quotient. In the second mode of proceeding no fraction of a farthing occurs in the result of the first division; and as in the final division such fraction would be omitted, fractions of a farthing are avoided altogether: they are introduced into the final quotient above only for the purpose of showing the complete similarity of the two results. When, however, the fraction exceeds half a farthing, it should be regarded as a whole farthing, rather than considered as of no account. The example upon which these remarks are made may be worked a little differently, and perhaps in the opinion of the learner a little more easily, as follows:—

| £     | s. | d.             | £     | s. | d.        |
|-------|----|----------------|-------|----|-----------|
| 7)237 | 13 | 5              | 2)237 | 13 | 5         |
| 2)33  | 19 | 0½...6 f. rem. | 7)118 | 16 | 8½        |
| £16   | 19 | 6¼ + ⅙ f.      | £16   | 19 | 6¼ + ¾ f. |

The learner is aware that  $\frac{3}{7}$  and  $\frac{6}{14}$  are the same.

| (3)   |    |                        | (4)   |    |                        |
|-------|----|------------------------|-------|----|------------------------|
| £     | s. | d.                     | £     | s. | d.                     |
| 7)562 | 18 | 6½                     | 6)106 | 19 | 3½                     |
| 5)80  | 8  | 4½...3 f. rem.         | 12)17 | 16 | 6½...1 f. rem.         |
| £16   | 1  | 8...10 f. rem. = 2½ d. | £1    | 9  | 8½...7 f. rem. = 1¾ d. |

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 (5) \quad 8 \overline{) 780 \quad 12 \quad 9\frac{1}{2}} \\
 \quad \quad 7 \overline{) 97 \quad 11 \quad 7} \dots 6 \text{ f. rem.} \\
 \quad \quad \quad 3 \overline{) 13 \quad 18 \quad 9\frac{1}{2}} \dots 22 \text{ f. rem.} \\
 \quad \quad \quad \underline{\text{£} 4 \quad 12 \quad 11} \dots 134 \text{ f. rem.} = 2\text{s. } 9\frac{1}{2}\text{d.}
 \end{array}$$

Although the remainders are not annexed to the answers in the book, yet the learner will do well to compute them, as in these examples. The fractional part of a farthing which completes the final quotient in Ex. 3 is  $\frac{19}{33}\text{f.}$ ; that which completes the final quotient in Ex. 4 is  $\frac{7}{72}\text{f.}$ ; and that belonging to Ex. 5 is  $\frac{134}{168}\text{f.}$ , which being more than half a farthing, £4 12s. 11½d. is more nearly the exact 168th part of £780 12s. 9½d. than £4 12s. 11d. The learner will also find it a profitable as well as an interesting exercise to *prove* all the results of division by *multiplying* each by the several factors already used as divisors, taking care however always to add in the final remainder.

| $\text{£} \quad \text{s.} \quad \text{d.}$                      | $\text{£} \quad \text{s.} \quad \text{d.}$ |
|-----------------------------------------------------------------|--------------------------------------------|
| (6) 273 $\overline{) 837 \quad 13 \quad 5\frac{1}{2}}$ (3 1 4½) |                                            |
| 819                                                             |                                            |
| <u>18</u>                                                       |                                            |
| 20                                                              |                                            |
| <u>373</u>                                                      |                                            |
| 273                                                             |                                            |
| <u>100</u>                                                      |                                            |
| 12                                                              |                                            |
| <u>1205</u>                                                     |                                            |
| 1092                                                            |                                            |
| <u>113</u>                                                      |                                            |
| 4                                                               |                                            |
| <u>454</u>                                                      |                                            |
| 273                                                             |                                            |
| <u>181</u> f. rem. = 3s. 9½d.                                   |                                            |

  

| <i>Proof.</i> |    |    |       |              |           |           |
|---------------|----|----|-------|--------------|-----------|-----------|
| £             | s. | d. |       | £            | s.        | d.        |
| 3             | 1  | 4½ | × 3 = | 9            | 4         | 0½        |
|               |    | 10 |       |              |           |           |
| 30            | 13 | 6½ | × 7 = | 214          | 14        | 9½        |
|               |    | 10 |       |              |           |           |
| 306           | 15 | 5  | × 2 = | 613          | 10        | 10        |
|               |    |    |       |              |           |           |
|               |    |    |       | 0            | 3         | 9½ rem    |
|               |    |    |       | <u>£ 837</u> | <u>13</u> | <u>5½</u> |

The complete quotient, or the exact 273d part of £837 13s. 5½d. is £3 1s. 4½d. and  $\frac{181}{273}\text{f.}$ ; that is, the 273d part of 181 farthings. As the fraction  $\frac{181}{273}$  is greater than  $\frac{1}{2}$  (for double the 181 is greater than 273), the farthing in the quotient may be made a halfpenny.

| <i>cwt. qr. lbs.</i>                                                                                         | <i>m. fur. po.</i>                                                                   |
|--------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| (7) $\begin{array}{r} 3 \overline{) 14 \ 1 \ 9} \\ 6 \overline{) 4 \ 3 \ 3} \\ \hline 0 \ 3 \ 5 \end{array}$ | (8) $\begin{array}{r} 11 \overline{) 823 \ 7 \ 21} \\ \hline 74 \ 7 \ 9 \end{array}$ |

2 oz....12 oz. rem.: hence the *complete* quotient is 3 qrs. 5 lbs.  $2\frac{1}{13}$  oz. \*

| <i>da. ho. min.</i>                                                                                                                                                                | <i>therefore</i>                                                                                                                                                                                                   |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (9) $\begin{array}{r} 5 \overline{) 128^\circ \ 45' \ 52''} \\ 5 \overline{) 25 \ 45 \ 10...2''} \\ 5 \overline{) 5 \ 9 \ 2} \\ \hline 1 \ 1 \ 48...52'' \text{ rem.} \end{array}$ | Or thus: $\begin{array}{r} 125 \overline{) 128^\circ \ 45' \ 52''} \\ \hline 3 \ 60 \\ \hline 225 \\ 125 \\ \hline 100 \\ 60 \\ \hline 6052 \\ 500 \\ \hline 1052 \\ 1000 \\ \hline 52'' \text{ rem.} \end{array}$ |

the *complete* quotient.

| <i>ac. roo. per.</i>                                                                                                                                  | <i>that is, the complete</i>                    |
|-------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------|
| (10) $\begin{array}{r} 2 \overline{) 315 \ 17 \ 38} \\ 7 \overline{) 157 \ 20 \ 49} \\ 8 \overline{) 22 \ 13 \ 15} \\ \hline 2 \ 19 \ 39 \end{array}$ | quotient is 2d. 19h. 39m. $26\frac{86}{113}$ s. |

26...86 sec. rem.: that is, the *complete* quotient is 2d. 19h. 39m.  $26\frac{86}{113}$ s.

| <i>ac. roo. per.</i>                                                                                                                                | <i>26 yds.</i>                                 |
|-----------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------|
| (11) $\begin{array}{r} 7 \overline{) 1784 \ 3 \ 32} \\ 5 \overline{) 254 \ 3 \ 38} \\ 3 \overline{) 50 \ 3 \ 39} \\ \hline 16 \ 3 \ 39 \end{array}$ | $\begin{array}{r} 26 \ 23 \ 3 \ 8 \end{array}$ |

The 26 yards is got by dividing  $181\frac{1}{2}$  by 7, which gives a quotient so near to 26, that 26 may be safely put down without risking the accuracy of the final result: the *remainders*, however, after feet, are suppressed, as they would be affected with error. But the best way of working this Example so as to get the complete quotient is by long division, as follows:—

\* The learner should accustom himself to state not only the complete remainder, but the complete quotient; and to examine whether or not the fractional correction exceeds half a unit, and if it do, to count it as a whole unit. For instance, the quotient above might with propriety be written 3 qrs. 5 lbs. 3 oz.; or, if it be thought worth while to notice a fraction of an ounce, it may be written, still more accurately, 3 qrs. 5 lbs.  $2\frac{1}{13}$  oz.

# 32 KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

|      | <i>ac.</i>       | <i>roo.</i> | <i>per.</i> | <i>ac.</i> | <i>roo.</i> | <i>per.</i> | <i>yd.</i> | <i>ft.</i> |
|------|------------------|-------------|-------------|------------|-------------|-------------|------------|------------|
| 105) | 1784             | 3           | 32          | (16        | 3           | 39          | 27         | 8          |
|      | 105              |             |             |            |             |             |            |            |
|      | 734              |             |             |            |             |             |            |            |
|      | 630              |             |             |            |             |             |            |            |
|      | 104              |             |             |            |             |             |            |            |
|      | 4                |             |             |            |             |             |            |            |
|      | 419              |             |             |            |             |             |            |            |
|      | 315              |             |             |            |             |             |            |            |
|      | 104              |             |             |            |             |             |            |            |
|      | 40               |             |             |            |             |             |            |            |
|      | 4192             |             |             |            |             |             |            |            |
|      | 315              |             |             |            |             |             |            |            |
|      | 1042             |             |             |            |             |             |            |            |
|      | 945              |             |             |            |             |             |            |            |
|      | 4)97             |             |             |            |             |             |            |            |
|      | 30 $\frac{1}{4}$ |             |             |            |             |             |            |            |

The fraction of a foot that completes the quotient here is  $\frac{213}{400}$ , which arises from dividing 4 times the remainder by 4 times the divisor: each is taken 4 times, because of the  $\frac{1}{4}$  in the remainder. The fraction may be replaced by  $\frac{1}{2}$ , as the bottom number is very nearly double the top one.

|                       |                           |
|-----------------------|---------------------------|
| for $\frac{1}{4}$ ... | 24 $\frac{1}{4}$          |
|                       | 2934 $\frac{1}{4}$        |
|                       | 210                       |
|                       | 834 $\frac{1}{4}$         |
|                       | 735                       |
|                       | 99 $\frac{1}{4}$          |
|                       | 9                         |
|                       | 893 $\frac{1}{4}$         |
|                       | 840                       |
|                       | 53 $\frac{1}{4}$ ft. rem. |

| (12) | £   | s. | d. | £                        | s. | d. |
|------|-----|----|----|--------------------------|----|----|
|      | 2   | 13 | 5  | 15                       | 16 | 9  |
|      | 20  |    |    | 20                       |    |    |
|      | 53  |    |    | 316                      |    |    |
|      | 12  |    |    | 12                       |    |    |
|      | 641 |    |    | 3801(5 $\frac{308}{611}$ |    |    |
|      |     |    |    | 3205                     |    |    |
|      |     |    |    | 596                      |    |    |

| (13) | £    | s. | d.              | £                            | s. | d.              |
|------|------|----|-----------------|------------------------------|----|-----------------|
|      | 7    | 3  | 4 $\frac{1}{4}$ | 89                           | 11 | 7 $\frac{1}{2}$ |
|      | 20   |    |                 | 20                           |    |                 |
|      | 143  |    |                 | 1791                         |    |                 |
|      | 12   |    |                 | 12                           |    |                 |
|      | 1720 |    |                 | 21499                        |    |                 |
|      | 4    |    |                 | 4                            |    |                 |
|      | 6881 |    |                 | 85998(12 $\frac{2128}{6881}$ |    |                 |
|      |      |    |                 | 6881                         |    |                 |
|      |      |    |                 | 17188                        |    |                 |
|      |      |    |                 | 13762                        |    |                 |
|      |      |    |                 | 3426                         |    |                 |

(14)

| £         | s. | d.              | £                                | s. | d. |
|-----------|----|-----------------|----------------------------------|----|----|
| 34        | 18 | 1 $\frac{3}{4}$ | 126                              | 7  | 0  |
| <u>20</u> |    |                 | <u>20</u>                        |    |    |
| 698       |    |                 | 2527                             |    |    |
| <u>12</u> |    |                 | <u>12</u>                        |    |    |
| 8377      |    |                 | 30324                            |    |    |
| <u>4</u>  |    |                 | <u>4</u>                         |    |    |
| 33511     |    |                 | 121296(3 $\frac{29763}{33511}$ ) |    |    |
|           |    |                 | <u>100533</u>                    |    |    |
|           |    |                 | <u>20763</u>                     |    |    |

(15)

| £         | s. | d.              | £                                | s. | d.              |
|-----------|----|-----------------|----------------------------------|----|-----------------|
| 47        | 6  | 9 $\frac{3}{4}$ | 321                              | 17 | 3 $\frac{3}{4}$ |
| <u>20</u> |    |                 | <u>20</u>                        |    |                 |
| 946       |    |                 | 6437                             |    |                 |
| <u>12</u> |    |                 | <u>12</u>                        |    |                 |
| 11361     |    |                 | 77247                            |    |                 |
| <u>4</u>  |    |                 | <u>4</u>                         |    |                 |
| 45447     |    |                 | 308991(6 $\frac{36309}{45447}$ ) |    |                 |
|           |    |                 | <u>272682</u>                    |    |                 |
|           |    |                 | <u>36309</u>                     |    |                 |

| cwt.      | qr. | lbs. | cwt.                        | qr. | lbs. |
|-----------|-----|------|-----------------------------|-----|------|
| (16)      | 5   | 1 14 | 73                          | 3   | 13   |
| <u>4</u>  |     |      | <u>4</u>                    |     |      |
| 21        |     |      | 295                         |     |      |
| <u>28</u> |     |      | <u>28</u>                   |     |      |
| 172       |     |      | 2363                        |     |      |
| <u>43</u> |     |      | <u>591</u>                  |     |      |
| 602       |     |      | 8273(13 $\frac{447}{602}$ ) |     |      |
|           |     |      | <u>602</u>                  |     |      |
|           |     |      | 2253                        |     |      |
|           |     |      | <u>1806</u>                 |     |      |
|           |     |      | <u>447</u>                  |     |      |

| d.        | h. | m.  | d.                            | h. | m. |
|-----------|----|-----|-------------------------------|----|----|
| (17)      | 5  | 1 2 | 78                            | 18 | 49 |
| <u>24</u> |    |     | <u>24</u>                     |    |    |
| 121       |    |     | 320                           |    |    |
| <u>60</u> |    |     | <u>167</u>                    |    |    |
| 7262      |    |     | 1890                          |    |    |
|           |    |     | <u>60</u>                     |    |    |
|           |    |     | 7262(11 $\frac{4519}{7262}$ ) |    |    |
|           |    |     | <u>7262</u>                   |    |    |
|           |    |     | 40829                         |    |    |
|           |    |     | <u>36310</u>                  |    |    |
|           |    |     | <u>4519</u>                   |    |    |

| (18)  | 13°       | 5' 18" | 79°                             | 13' 46" |
|-------|-----------|--------|---------------------------------|---------|
|       | <u>60</u> |        | <u>60</u>                       |         |
|       | 785       |        | 4753                            |         |
|       | <u>60</u> |        | <u>60</u>                       |         |
| 47118 |           |        | 285226(6 $\frac{2518}{47118}$ ) |         |
|       |           |        | <u>282708</u>                   |         |
|       |           |        | <u>2518</u>                     |         |

| (19) | qrs.      | lbs. | t.                                | cwt. | qrs. | lbs. |
|------|-----------|------|-----------------------------------|------|------|------|
|      | 3         | 17   | 2                                 | 13   | 0    | 5    |
|      | <u>28</u> |      | <u>20</u>                         |      |      |      |
|      | 101       |      | 53                                |      |      |      |
|      |           |      | <u>4</u>                          |      |      |      |
|      |           |      | 212                               |      |      |      |
|      |           |      | <u>28</u>                         |      |      |      |
|      |           |      | 1701                              |      |      |      |
|      |           |      | <u>424</u>                        |      |      |      |
|      |           |      | 1015941(58 $\frac{22}{1015941}$ ) |      |      |      |
|      |           |      | <u>505</u>                        |      |      |      |
|      |           |      | 891                               |      |      |      |
|      |           |      | <u>808</u>                        |      |      |      |
|      |           |      | <u>83</u>                         |      |      |      |

| (20) | £          | s.        | d.                               |
|------|------------|-----------|----------------------------------|
|      | 3          | 17        | 10 $\frac{1}{2}$                 |
|      |            |           | <u>12</u> number of oz. in 1 lb. |
|      | <u>£46</u> | <u>14</u> | <u>6</u>                         |

$$(21) \quad 607)3288192(5417l. \quad 2s. \quad 4\frac{1}{2}d. + \frac{275}{207}f.$$

$$\begin{array}{r} 3035 \\ \underline{2531} \\ 2428 \\ \underline{1039} \\ 607 \\ \underline{4322} \\ 4249 \\ \underline{73} \end{array} \quad \begin{array}{r} 73 \\ \underline{20} \\ 1460 \\ \underline{1214} \\ 246 \\ \underline{12} \\ 2952 \\ \underline{2428} \\ 524 \\ \underline{4} \\ 2096 \\ \underline{1821} \\ 275 \end{array}$$

$$(22) \quad \frac{120}{3} = 40 \text{ number of pence at 3 a penny.}$$

$$\frac{120}{2} = 60 \quad \text{,,} \quad \text{at 2 a penny.}$$

100 number of pence charged.

$$\frac{240}{5} = 48 \text{ number of two-pences paid, at 5 for 2d.}$$

$$48 \times 2 = 96 \text{ number of pence paid.}$$

$$\underline{4} \text{ number of pence saved.}$$

$$(23) \quad \text{Received in gold and silver.}$$

$$\text{Received in copper.}$$

$$\begin{array}{l} \text{£4 } 10 \quad 0 = 4 \text{ sovs. and } \frac{1}{2} \\ \quad \quad 5 \quad 0 = 1 \text{ crown.} \end{array}$$

$$\frac{88}{8} l. = 11 \quad 0 = 88 \text{ half-crowns.} \quad \frac{665}{12} s. = 55s. \quad 5d. = 2 \quad 15 \quad 5$$

$$\frac{992}{20} l. = 49 \quad 12 \quad 0 = 992 \text{ shillings.} \quad \frac{667}{24} s. = 27s. \quad 9\frac{1}{2}d. = 1 \quad 7 \quad 9\frac{1}{2}$$

$$\frac{842}{40} l. = 21 \quad 1 \quad 0 = 842 \text{ sixpences.} \quad \frac{25}{4} d. = \quad \quad \quad 6\frac{1}{4}$$

$$\frac{142}{3} s. = 2 \quad 7 \quad 4 = 142 \text{ fourpenny pieces.} \quad \text{£4 } \underline{\underline{3 \quad 8\frac{3}{4}}}$$

$$\frac{5}{4} s. = \quad 1 \quad 3 = 5 \text{ threepenny pieces.}$$

$$\underline{\underline{88 \quad 16 \quad 7}} \text{ amount in gold and silver.}$$

$$\underline{\underline{4 \quad 3 \quad 8\frac{3}{4}}} \text{ amount in copper.}$$

$$4)93 \quad 0 \quad 3\frac{3}{4} \text{ total amount.}$$

$$\underline{\underline{23 \quad 5 \quad 0\frac{3}{4}}} + \frac{1}{4} f. \text{ share of each.}$$

$$\begin{array}{r}
 \text{oz. dwt. grs.} \\
 (24) \quad 12 \overline{) 9 \, 18 \, 20} \\
 \underline{16 \, 13 \frac{6}{12}}
 \end{array}$$

$$\begin{array}{r}
 (25) \quad 312500 \\
 \underline{20} \\
 6250000 \\
 \underline{24} \\
 25000000 \\
 \underline{1250}
 \end{array}$$

$$\begin{array}{r}
 113)150000000(1327433 \frac{1}{2} \text{ sovs., and} \\
 \underline{113} \qquad \qquad \qquad 14 \frac{1}{2} \text{ grains over.}
 \end{array}$$

$$\begin{array}{r}
 370 \\
 339 \\
 \hline
 310 \\
 226 \\
 \hline
 840 \\
 791 \\
 \hline
 490 \\
 452 \\
 \hline
 380 \\
 339 \\
 \hline
 410 \\
 339 \\
 \hline
 71 \\
 56 \frac{1}{2} \\
 \hline
 14 \frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 (26) \quad 312500 \\
 \underline{3} \\
 \text{at } £3 \text{ per oz. } \quad £937500
 \end{array}$$

$$\begin{array}{r}
 12)3125000 \\
 2,0 \overline{) 26041 \, 6s. \, 8d.} \\
 \text{at } 10d. \quad \underline{£13020 \, 16s. \, 8d.}
 \end{array}$$

$$\begin{array}{r}
 2)312500 \\
 12)156250 \\
 2,0 \overline{) 13020 \, 10d.} \\
 \text{at } \frac{1}{2}d. \quad \underline{£651 \, 0s. \, 10d.}
 \end{array}$$

$$\begin{array}{r}
 2,0 \overline{) 31250,0} \\
 15625 \\
 \underline{17} \\
 109375 \\
 15625 \\
 \hline
 £265625 \text{ at } 17s. \text{ per oz.} \\
 937500 \text{ at } £3. \\
 13020 \, 16s. \, 8d. \text{ at } 10d. \\
 651 \, 0s. \, 10d. \text{ at } \frac{1}{2}d. \\
 \hline
 \underline{£1216796 \, 17 \, 6 \text{ at } £3 \, 17s. \, 10 \frac{1}{2}d.}
 \end{array}$$

Example 26 may be otherwise worked by using as multipliers the factors of 312500, namely,  $10 \times 10 \times 5 \times 5 \times 5 \times 5$ ; as follows:—

| £        | s. | d.              |
|----------|----|-----------------|
| 3        | 17 | $10\frac{1}{2}$ |
| <hr/>    |    |                 |
| 38       | 18 | 9               |
| <hr/>    |    |                 |
| 389      | 7  | 6               |
| <hr/>    |    |                 |
| 1946     | 17 | 6               |
| <hr/>    |    |                 |
| 9734     | 7  | 6               |
| <hr/>    |    |                 |
| 48671    | 17 | 6               |
| <hr/>    |    |                 |
| 243359   | 7  | 6               |
| <hr/>    |    |                 |
| £1216796 | 17 | 6               |

(27)

| s.             | d. |
|----------------|----|
| 4              | 8  |
| <hr/>          |    |
| 1              | 17 |
| <hr/>          |    |
| £14            | 18 |
| <hr/>          |    |
| £14            | 18 |
| <hr/>          |    |
| 8 price of the |    |
| 64 lbs.        |    |

But the best way of proceeding would be first to find the value of the 312500 oz. at 2s.  $1\frac{1}{2}$ d. per oz., and then to subtract the result from the value at £4 per oz. As  $1\frac{1}{2}$ d. is the eighth part of 1s., the value at 2s.  $1\frac{1}{2}$ d. will be obtained by multiplying the number of ounces by 2, and then adding one-eighth of that number to the product, thus:—

|             |                                        |
|-------------|----------------------------------------|
| 8           | 312500                                 |
| <hr/>       |                                        |
| 2           |                                        |
| <hr/>       |                                        |
| 625000      |                                        |
| 39062       | 6                                      |
| <hr/>       |                                        |
| 20          | 664062                                 |
| <hr/>       |                                        |
| £33203      | 2s. 6d. val. at 2s. $1\frac{1}{2}$ d.  |
| <hr/>       |                                        |
| 312500      |                                        |
| <hr/>       |                                        |
| 4           |                                        |
| <hr/>       |                                        |
| £1250000    | val. at £4.                            |
| <hr/>       |                                        |
| sub. £33203 | 2s. 6d. at 2s. $1\frac{1}{2}$ d.       |
| <hr/>       |                                        |
| £1216796    | 17s. 6d. at £3 17s. $10\frac{1}{2}$ d. |

| s.    | d.  |                                                             |
|-------|-----|-------------------------------------------------------------|
| 4     | 4   | 64                                                          |
| <hr/> |     | 42                                                          |
| 1     | 10  | 4                                                           |
| <hr/> |     | 6                                                           |
| <hr/> |     | 106 the whole No. of lbs                                    |
| <hr/> |     |                                                             |
| £9    | 2   | 0 price of the 42 lbs.                                      |
| <hr/> |     |                                                             |
| £14   | 18  | 8                                                           |
| <hr/> |     |                                                             |
| £24   | 0   | 8 price of the whole 106 lbs.                               |
| <hr/> |     |                                                             |
| 20    |     |                                                             |
| 106   | 480 | (4s. $6\frac{1}{4}$ d. + $\frac{10}{100}$ f. price of 1 lb. |
| <hr/> |     |                                                             |
| 424   |     |                                                             |
| <hr/> |     |                                                             |
| 56    |     |                                                             |
| <hr/> |     |                                                             |
| 12    |     |                                                             |
| <hr/> |     |                                                             |
| 680   |     |                                                             |
| <hr/> |     |                                                             |
| 636   |     |                                                             |
| <hr/> |     |                                                             |
| 44    |     |                                                             |
| <hr/> |     |                                                             |
| 4     |     |                                                             |
| <hr/> |     |                                                             |
| 176   |     |                                                             |
| <hr/> |     |                                                             |
| 106   |     |                                                             |
| <hr/> |     |                                                             |
| 70    |     |                                                             |
| <hr/> |     |                                                             |



(28) 123

$$\begin{array}{r}
 400000 \\
 12 \overline{) 49200000} \text{ grains} \\
 \underline{2) 4100000} \\
 2,0 \overline{) 205000,0} \text{ dwt.} \\
 12 \overline{) 102500} \text{ oz.} \\
 \text{weight } \underline{8541 \text{ lbs. 8 oz.}}
 \end{array}$$

$$\begin{array}{r}
 4 \overline{) 400000} \\
 4 \overline{) 100000} \\
 12 \overline{) 25000} \text{ inches} \\
 202 \text{ ft. } \overline{) 2083 \text{ ft. 4 in.}} (10 \text{ times, and} \\
 \underline{202} \quad \quad \quad 63 \text{ ft. 4 in. over} \\
 63
 \end{array}$$

(29) The money he must have must evidently be 2831 times 13s. 2½d., which may be found as follows:—

$$\begin{array}{r}
 2831 \\
 \underline{13} \\
 8493 \\
 \underline{2831} \\
 36803s. \\
 \underline{589s. 9\frac{1}{2}d.} \\
 2,0 \overline{) 3739,2s. 9\frac{1}{2}d.} \\
 \underline{\pounds 1869 \text{ 12s. 9}\frac{1}{2}d.}
 \end{array}$$

$$\begin{array}{r}
 \text{gr.} \\
 (30) \quad 129\frac{1}{2} \text{ wt. of guinea} \\
 \underline{123\frac{1}{4}} \text{ „ of sovereign} \\
 6\frac{1}{4} \text{ difference}
 \end{array}$$

$$\begin{array}{r}
 4 \overline{) 100000} \\
 \underline{6\frac{1}{4}} \\
 600000 \\
 \underline{25000} \\
 2 \overline{) 625000} \text{ gr.} \\
 12 \overline{) 312500} \\
 2,0 \overline{) 2604,1} \text{ dwt. 16 gr.} \\
 12 \overline{) 1302} \text{ oz. 1 dwt.} \\
 \underline{108 \text{ lbs. 6 oz. 1 dwt. 16 gr.}}
 \end{array}$$

(31) 4) 2387881

$$\begin{array}{r}
 123\frac{1}{4} \\
 \underline{7163643} \\
 28654572 \\
 \underline{596970 \text{ for } \frac{1}{4}} \\
 3 \overline{) 294306333} \\
 \underline{8) 98102111} \\
 2,0 \overline{) 1226276,3...21} \text{ gr.} \\
 \underline{12) 613138...3} \text{ dwt.} \\
 \text{total weight } \underline{51094 \text{ lbs. 10 oz. 3 dwt.}} \\
 21 \text{ gr.}
 \end{array}$$

$$\begin{array}{r}
 2387881 \\
 \underline{113} \\
 7163643 \\
 \underline{26266691} \\
 3 \overline{) 269830553} \\
 \underline{8) 89943517...2} \text{ gr.} \\
 2,0 \overline{) 1124293,9...17} \text{ gr.} \\
 \underline{12) 562146...19} \text{ dwt.} \\
 \text{pure gold } \underline{46845 \text{ lbs. 6 oz. 19 dwt.}} \\
 17 \text{ gr.}
 \end{array}$$

As it is customary to take 123¼ grains as the weight of a sovereign, this number has been used in the preceding work; but the Mint weight really exceeds this by a small fraction of a grain; and, in computing the weight of so many sovereigns as in the Example, the neglect of

this small fraction makes a very sensible difference in the result. A reference to the Table, at p. 38 of the Arithmetic, will show that the exact weight of a sovereign is  $123\frac{1}{2}\frac{1}{2}$  grains, where  $\frac{1}{2}\frac{1}{2}$  exceeds  $\frac{1}{2}$ , because 4 times 171 exceeds 623; by replacing the fraction by  $\frac{1}{2}$ , we replace 171 by  $155\frac{1}{2}$ , 4 times which is exactly 623; so that the weight of each sovereign is made too little by  $\frac{15\frac{1}{2}}{623}$  gr., that is, by  $\frac{1}{2}\frac{1}{2}$  gr. We may therefore make the above weight strictly accurate by multiplying 2387881 by 61, and dividing the product by 2492; the quotient will be the additional weight, in grains, to be added to the weight determined above. The learner will find this additional weight to be 10 lbs. 1 oz. 15 dwt. 11 gr., without counting fractions of a grain.

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### FRACTIONS.

*Reduction to Improper Fractions and Mixed Numbers,*  
page 70.

$$\begin{array}{lll} \text{(1)} & \text{(2)} & \text{(3)} \\ 7\frac{2}{3} = \frac{21+2}{3} = \frac{23}{3} & 17\frac{3}{7} = \frac{119+3}{7} = \frac{122}{7} & 11\frac{1}{12} = 11\frac{1}{12} \end{array}$$

$$\begin{array}{ll} \text{(4)} & \text{(5)} \\ 17 \overline{) 2075} (122\frac{1}{17} & \begin{array}{r} 238 \\ 27 \\ \hline 1666 \end{array} \therefore 238\frac{18}{27} = 218\frac{2}{3} \\ \underline{17} & \begin{array}{r} 476 \\ \hline 6426 \\ \hline 19 \\ \hline 6445 \end{array} \end{array}$$

$$\begin{array}{ll} \text{(6)} & \text{(7)} \\ 123 \overline{) 31872} (259\frac{15}{123} = 218\frac{2}{3} & \begin{array}{r} 2016 \\ 19 \\ \hline 18144 \\ 2016 \end{array} \therefore 2016\frac{27}{19} = 211\frac{18}{19} \\ \underline{246} & \begin{array}{r} 38304 \\ \hline 27 \\ \hline 38331 \end{array} \\ \underline{727} & \\ \underline{615} & \\ \underline{1122} & \\ \underline{1107} & \\ \underline{15} & \end{array}$$

$$\begin{array}{l} \text{(8)} \quad 317 \overline{) 12731} (40\frac{51}{317} \therefore 40\frac{51}{317} \\ \underline{1268} \\ 51 \end{array}$$

\* The three dots  $\therefore$  stand for the word *therefore*.

*Reduction of Fractions to Common Denominators,  
pages 72, 74.*

$$\begin{array}{l} (1) \\ 1 \times 3 \times 5 = 15 \\ 2 \times 2 \times 5 = 20 \\ 1 \times 2 \times 3 = 6 \\ 2 \times 3 \times 5 = 30, \text{ com. denom.} \end{array} \left. \begin{array}{l} \text{numerators of} \\ \text{changed fractions} \end{array} \right\}$$

$\therefore$  the fractions in a com. den. are

$$\frac{15}{30}, \quad \frac{20}{30}, \quad \frac{6}{30}$$

$$\begin{array}{l} (2) \\ 3 \times 5 \times 3 = 45 \\ 2 \times 4 \times 3 = 24 \\ 1 \times 4 \times 5 = 20 \\ 4 \times 5 \times 3 = 60, \text{ com. denom.} \end{array} \left. \begin{array}{l} \text{numerators} \end{array} \right\}$$

$\therefore$  the fractions are

$$\frac{45}{60}, \quad \frac{24}{60}, \quad \frac{20}{60}$$

$$\begin{array}{l} (3) \\ 3 \times 7 \times 9 = 189 \\ 2 \times 5 \times 9 = 90 \\ 4 \times 5 \times 7 = 140 \\ 5 \times 7 \times 9 = 315, \text{ com. den.} \end{array} \left. \begin{array}{l} \text{numerators} \end{array} \right\}$$

$\therefore$  the fractions are

$$\frac{189}{315}, \quad \frac{90}{315}, \quad \frac{140}{315}$$

$$\begin{array}{l} (4) \\ 3 \times 6 \times 3 = 54 \\ 5 \times 8 \times 3 = 120 \\ 7 \times 8 \times 6 = 336 \\ 8 \times 6 \times 3 = 144, \text{ com. den.} \end{array} \left. \begin{array}{l} \text{numerators} \end{array} \right\}$$

$\therefore$  the fractions are

$$\frac{54}{144}, \quad \frac{120}{144}, \quad \frac{336}{144}$$

$$\begin{array}{l} (5) \\ 7 \times 11 \times 7 = 539 \\ 3 \times 9 \times 7 = 189 \\ 5 \times 9 \times 11 = 495 \\ 9 \times 11 \times 7 = 693, \text{ com. den.} \end{array} \left. \begin{array}{l} \text{numerators} \end{array} \right\}$$

$\therefore$  the fractions are

$$\frac{539}{693}, \quad \frac{189}{693}, \quad \frac{495}{693}$$

$$\begin{array}{l} (6) \\ 2 \times 10 \times 12 = 240 \\ 9 \times 13 \times 12 = 1404 \\ 11 \times 13 \times 10 = 1430 \\ 13 \times 10 \times 12 = 1560, \text{ com. den.} \end{array} \left. \begin{array}{l} \text{numerators} \end{array} \right\}$$

$\therefore$  the fractions are

$$\frac{240}{1560}, \quad \frac{1404}{1560}, \quad \frac{1430}{1560}$$

$$\begin{array}{l} (7) \\ 3 \times 5 \times 7 \times 8 = 840 \\ 2 \times 4 \times 7 \times 8 = 448 \\ 6 \times 4 \times 5 \times 8 = 960 \\ 1 \times 4 \times 5 \times 7 = 140 \\ 4 \times 5 \times 7 \times 8 = 1120, \text{ com. den.} \end{array} \left. \begin{array}{l} \text{numerators} \end{array} \right\}$$

$\therefore$  the fractions are

$$\frac{840}{1120}, \quad \frac{448}{1120}, \quad \frac{960}{1120}, \quad \frac{140}{1120}$$

$$\begin{array}{l} (8) \\ 4 \times 8 \times 6 \times 2 = 384 \\ 3 \times 5 \times 6 \times 2 = 180 \\ 5 \times 5 \times 8 \times 2 = 400 \\ 1 \times 5 \times 8 \times 6 = 240 \\ 5 \times 8 \times 6 \times 2 = 480, \text{ com. den.} \end{array} \left. \begin{array}{l} \text{numerators} \end{array} \right\}$$

$\therefore$  the fractions are

$$\frac{384}{480}, \quad \frac{180}{480}, \quad \frac{400}{480}, \quad \frac{240}{480}$$

(9)

$$\left. \begin{array}{l} 6 \times 5 \times 3 \times 4 = 360 \\ 3 \times 7 \times 3 \times 4 = 252 \\ 4 \times 7 \times 5 \times 4 = 560 \\ 3 \times 7 \times 5 \times 3 = 315 \\ 7 \times 5 \times 3 \times 4 = 420, \text{ com. den.} \end{array} \right\} \text{numerators}$$

 $\therefore$  the fractions are

$$\frac{360}{420}, \quad \frac{252}{420}, \quad \frac{560}{420}, \quad \frac{315}{420}$$

(10)

$$\left. \begin{array}{l} 5 \times 11 \times 7 \times 4 = 1540 \\ 3 \times 9 \times 7 \times 4 = 756 \\ 2 \times 9 \times 11 \times 4 = 792 \\ 1 \times 9 \times 11 \times 7 = 693 \\ 9 \times 11 \times 7 \times 4 = 2772, \text{ com. den.} \end{array} \right\} \text{nume-} \\ \text{rators}$$

 $\therefore$  the fractions are

$$\frac{1540}{2772}, \quad \frac{756}{2772}, \quad \frac{792}{2772}, \quad \frac{693}{2772}$$

(11)

$$\left. \begin{array}{l} 1 \times 7 \times 2 \times 9 = 126 \\ 3 \times 2 \times 2 \times 9 = 108 \\ 7 \times 2 \times 7 \times 9 = 882 \\ 5 \times 2 \times 7 \times 2 = 140 \\ 2 \times 7 \times 2 \times 9 = 252, \text{ com. den.} \end{array} \right\} \text{nume-} \\ \text{rators}$$

 $\therefore$  the fractions are

$$\frac{126}{252}, \quad \frac{108}{252}, \quad \frac{882}{252}, \quad \frac{140}{252}$$

(12)

$$\left. \begin{array}{l} 3 \times 5 \times 7 \times 9 = 945 \\ 3 \times 4 \times 7 \times 9 = 756 \\ 1 \times 4 \times 5 \times 9 = 180 \\ 5 \times 4 \times 5 \times 7 = 700 \\ 4 \times 5 \times 7 \times 9 = 1260, \text{ com. den.} \end{array} \right\} \text{numerators}$$

 $\therefore$  the fractions are

$$\frac{945}{1260}, \quad \frac{756}{1260}, \quad \frac{180}{1260}, \quad \frac{700}{1260}$$

(13) Here the fraction  $\frac{4}{16}$  may be replaced by  $\frac{1}{4}$ , after which the only differing denominators will be 4 and 8, which become equal upon multiplying the former by 2; therefore, multiplying *both* terms of the first fraction by 2, the three fractions, in a common denominator, are

$$\frac{6}{8}, \quad \frac{5}{8}, \quad \frac{2}{8}$$

(14) The first and third fractions are brought to a common denominator simply by multiplying the terms of the first by 3; the only differing denominators will then be 9, and 5, which become equal by multiplying the former by 5 and the latter by 9; the steps are therefore as follow:—

$$\frac{2}{3}, \quad \frac{3}{5}, \quad \frac{7}{9} = \frac{6}{9}, \quad \frac{3}{5}, \quad \frac{7}{9} = \frac{30}{45}, \quad \frac{27}{45}, \quad \frac{35}{45}$$

the terms of the last fraction being each multiplied by 5.

(15) The terms of the second fraction are each divisible by 3, so that the fractions may be written

$$\frac{5}{7}, \quad \frac{4}{3}, \quad \frac{13}{24}$$

The second of these is brought to the same denominator as the third by multiplying its terms by 8, so that the three fractions are

$\frac{5}{7}, \frac{32}{24}, \frac{13}{3}$ ; and if the terms of the first of these be multiplied by 24, and the terms of each of the others by 7, it is plain that  $24 \times 7$  or 168 will be a common denominator of all; therefore the changed fractions are

$$\frac{120}{168}, \quad \frac{224}{168}, \quad \frac{91}{168}$$

(16) As in each fraction the denominator is divisible by the numerator, the fractions may be written  $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ ; and these are brought to a common denominator by multiplying each of the terms of the first two by 3.  $\therefore$  the changed fractions are

$$\frac{3}{9}, \quad \frac{3}{9}, \quad \frac{1}{9}$$

(17) By looking at the denominators of the second and third fractions, it is readily seen that  $9 \times 12 = 108$  and  $54 \times 2 = 108$ ; therefore if the terms of the second fraction be each multiplied by 12, and those of the third by 2, the three fractions in a common denominator will be

$$\frac{5}{108}, \quad \frac{24}{108}, \quad \frac{22}{108}$$

(18) Dividing the terms of the last fraction by 4, the three become  $\frac{5}{25}, \frac{6}{25}, \frac{2}{25}$ ; therefore, multiplying the terms of the first by 5, the fractions in a common denominator are

$$\frac{35}{25}, \quad \frac{4}{25}, \quad \frac{2}{25}$$

(19) These fractions are the same as  $\frac{2}{9}, \frac{5}{9}, \frac{7}{18}, \frac{1}{2}$ ; so that by multiplying the terms of the first two by 2, and those of the last by 9, the fractions in a common denominator become

$$\frac{4}{18}, \quad \frac{10}{18}, \quad \frac{7}{18}, \quad \frac{9}{18}$$

(20) Seeing that the denominator of the third fraction is twice that of the first, we multiply the terms of the first by 2; the fractions then become

$$\frac{2}{4}, \quad \frac{1}{3}, \quad \frac{1}{4}, \quad \frac{1}{5}$$

where the differing denominators are 4, 3 and 5; and these will be made equal by multiplying each by the product of the others; so that the fractions in a common denominator are

$$\frac{30}{60}, \quad \frac{20}{60}, \quad \frac{15}{60}, \quad \frac{12}{60}$$

(21) This Example is worked in the Book (Art. 54).

(22) These fractions are the same as

$$\frac{2}{9}, \quad \frac{1}{3}, \quad \frac{5}{12}, \quad \frac{7}{2} = \frac{2}{9}, \quad \frac{3}{9}, \quad \frac{5}{12}, \quad \frac{42}{12}$$

and since  $9 \times 4$  and  $12 \times 3$  give the same product, namely, 36, the fractions will have this common denominator if the terms of the first two be each multiplied by 4, and the terms of each of the others by 3; hence the changed fractions are

$$\frac{8}{36}, \quad \frac{12}{36}, \quad \frac{15}{36}, \quad \frac{126}{36}$$

(23) These fractions are evidently the same as  $2\frac{2}{12}, \frac{5}{12}, \frac{8}{12}, \frac{3}{12}$ , the terms of the first being multiplied by 2, those of the third by 4, and those of the fourth by 3.

(24) Multiplying the terms of the first fraction by 3, and those of the second by 5, to make the denominators each like that of the last fraction, the given fractions become  $3\frac{5}{15}, \frac{19}{15}, \frac{5}{25}, \frac{4}{15}$ ; and since  $25 = 5 \times 5$  and  $15 = 5 \times 3$ , we have only to multiply the terms of the third fraction by 5, and those of the others by 3, to get the forms  $3\frac{5}{15}, \frac{59}{15}, \frac{9}{15}, \frac{20}{15}$ .



### *Addition of Fractions, page 76.*

$$(1) \quad \frac{1}{4} + \frac{2}{5} + \frac{3}{7} = \frac{35}{140} + \frac{56}{140} + \frac{60}{140} = \frac{35+56+60}{140} = \frac{151}{140} = 1\frac{11}{140}.$$

$$(2) \quad \frac{3}{4 \times 2} + \frac{1}{3 \times 2} + \frac{4}{5} = \frac{3 \times 3}{4 \times 2 \times 3} + \frac{4}{3 \times 2 \times 4} + \frac{4}{5} = \frac{9}{24} + \frac{4}{24} + \frac{4}{5} = \frac{13}{24} + \frac{4}{5} \\ = \frac{13 \times 5}{24 \times 5} + \frac{4 \times 24}{5 \times 24} = \frac{65+96}{120} = 1\frac{41}{120}.$$

$$(3) \quad \frac{5}{6 \times 2} + \frac{3}{5 \times 2} + 1\frac{1}{7} = \frac{5 \times 5}{6 \times 2 \times 5} + \frac{3 \times 6}{5 \times 2 \times 6} + 1\frac{1}{7} = \frac{25+18}{60} + 1\frac{1}{7} \\ = \frac{43}{60} + 1\frac{1}{7} = \frac{43 \times 7}{60 \times 7} + 1\frac{60}{7 \times 60} = 1\frac{361}{420}.$$

$$(4) \quad \frac{2}{9} + \frac{5}{8} = \frac{16+45}{72} = \frac{61}{72}. \quad \therefore \frac{4}{18} + \frac{5}{8} + 6 = 6\frac{61}{72}.$$

$$(5) \quad \frac{7}{9} + \frac{2}{3} + \frac{1}{2} = \frac{7}{9} + \frac{6}{9} + \frac{1}{2} = \frac{13}{9} + \frac{1}{2} = 1\frac{4}{9} + \frac{1}{2} = 1\frac{8}{18} + \frac{9}{18} = 1\frac{17}{18}. \\ \therefore 2\frac{7}{9} + 3\frac{2}{3} + \frac{1}{2} = 6\frac{17}{18}.$$

$$(6) \quad \frac{1}{2} + \frac{11}{12} + \frac{3}{8} = \frac{6}{12} + \frac{11}{12} + \frac{3}{8} = \frac{17}{12} + \frac{3}{8} = \frac{34}{24} + \frac{9}{24} = \frac{43}{24} = 1\frac{19}{24}.$$

$$\therefore 8\frac{1}{2} + 5\frac{11}{12} + \frac{3}{8} = 14\frac{19}{24}.$$

$$(7) \quad \frac{7}{24} + \frac{5}{9} = \frac{7}{8 \times 3} + \frac{5}{3 \times 3} = \frac{7 \times 3}{8 \times 3 \times 3} + \frac{5 \times 8}{3 \times 3 \times 8} = \frac{21 + 40}{72} = \frac{61}{72}.$$

$$\therefore \frac{15}{3} + \frac{7}{24} + 3\frac{5}{9} = 8\frac{61}{72}.$$

$$(8) \quad \frac{4}{54} + \frac{7}{54} + \frac{2}{5} = \frac{11}{54} + \frac{2}{5} = \frac{11 \times 5}{54 \times 5} + \frac{2 \times 54}{5 \times 54} = \frac{55 + 108}{270} = \frac{163}{270}.$$

$$\therefore \frac{2}{27} + \frac{7}{54} + 1\frac{2}{5} = 1\frac{163}{270}.$$

$$(9) \quad \frac{17}{5 \times 5} + \frac{3}{5 \times 2} + \frac{13}{8 \times 2} = \frac{17 \times 8 \times 2}{5 \times 5 \times 8 \times 2} + \frac{3 \times 5 \times 8}{5 \times 2 \times 5 \times 8} + \frac{13 \times 5 \times 5}{8 \times 2 \times 5 \times 5} =$$

$$\frac{272 + 120 + 325}{400} = \frac{717}{400} = 1\frac{317}{400}.$$

$$\therefore 2\frac{17}{25} + 1\frac{3}{10} + \frac{13}{16} = 4\frac{317}{400}.$$

$$(10) \quad \frac{9}{2 \times 7} + \frac{11}{9 \times 7} + \frac{2}{3 \times 5} + \frac{7}{4 \times 5} = \frac{9 \times 9}{2 \times 7 \times 9} + \frac{11 \times 2}{9 \times 7 \times 2} + \frac{2 \times 4}{3 \times 5 \times 4} + \frac{7 \times 3}{4 \times 5 \times 3}$$

$$= \frac{81}{126} + \frac{22}{60} = \frac{103}{126} + \frac{29}{60}$$

$$= \frac{103}{21 \times 6} + \frac{29}{10 \times 6} = \frac{103 \times 10}{21 \times 6 \times 10} + \frac{29 \times 21}{10 \times 6 \times 21}$$

$$= \frac{1030 + 609}{1260} = \frac{1639}{1260} = 1\frac{379}{1260}.$$

$$\therefore 1\frac{9}{14} + \frac{11}{63} + \frac{2}{15} + 3\frac{7}{20} = 5\frac{379}{1260}.$$

$$(11) \quad \frac{14}{3 \times 5} + \frac{39}{8 \times 5} = \frac{14 \times 8}{3 \times 5 \times 8} + \frac{39 \times 3}{8 \times 5 \times 3} = \frac{112 + 117}{120} = \frac{229}{120} = 1\frac{109}{120}.$$

$$\text{And } \frac{11}{4 \times 3} + \frac{26}{9 \times 3} = \frac{11 \times 9}{9 \times 3 \times 9} + \frac{26 \times 4}{9 \times 3 \times 4} = \frac{99 + 104}{108} = \frac{203}{108} = 1\frac{95}{108}.$$

$$\text{Also } \frac{109}{120} + \frac{95}{108} = \frac{109}{10 \times 12} + \frac{95}{9 \times 12} = \frac{109 \times 9}{10 \times 12 \times 9} + \frac{95 \times 10}{9 \times 12 \times 10} = \frac{981 + 950}{1080}$$

$$= \frac{1931}{1080} = 1\frac{851}{1080}. \quad \therefore \frac{14}{15} + \frac{11}{12} + \frac{26}{27} + \frac{39}{40} = 3\frac{851}{1080}.$$

(12) The fractions are  $1\frac{8}{27} + \frac{1}{6} + 5\frac{9}{24} + \frac{5}{12}$ ; or, multiplying the terms of the second by 2, and adding to the third, they are

$$1\frac{8}{27} + \frac{7}{12} + 5\frac{9}{24}. \text{ Now } \frac{8}{9 \times 3} + \frac{7}{4 \times 3} = \frac{8 \times 4}{9 \times 3 \times 4} + \frac{7 \times 9}{4 \times 3 \times 9} = \frac{32 + 63}{108} = \frac{95}{108}.$$

$$\text{And } \frac{95}{108} + \frac{9}{24} = \frac{95}{27 \times 4} + \frac{9}{7 \times 4} = \frac{95 \times 7}{27 \times 4 \times 7} + \frac{9 \times 27}{7 \times 4 \times 27} = \frac{665 + 243}{756} = \frac{908}{756}$$

$$= 1\frac{152}{756} = 1\frac{38}{189}. \therefore 1\frac{8}{27} + \frac{1}{6} + 5\frac{9}{24} + \frac{5}{12} = 7\frac{38}{189}.$$

$$(13) \frac{19}{9 \times 7} + \frac{17}{5 \times 7} = \frac{19 \times 5}{9 \times 7 \times 5} + \frac{17 \times 9}{5 \times 7 \times 9} = \frac{95 + 153}{315} = \frac{248}{315}.$$

$$\text{And } \frac{11}{2 \times 13} + \frac{5}{3 \times 13} = \frac{11 \times 3}{2 \times 13 \times 3} + \frac{5 \times 2}{3 \times 13 \times 2} = \frac{33 + 10}{78} = \frac{43}{78}.$$

$$\text{Then } \frac{248}{315} + \frac{43}{78} = \frac{248}{105 \times 3} + \frac{43}{26 \times 3} = \frac{248 \times 26}{105 \times 3 \times 26} + \frac{43 \times 105}{26 \times 3 \times 105}$$

$$= \frac{6448 + 4515}{8190} = \frac{10963}{8190} = 1\frac{2773}{8190}, \text{ the sum required.}$$

(14) Here no two denominators have a factor in common; therefore applying the rule to the fractions  $\frac{13}{35} + \frac{7}{27} + \frac{8}{11} + \frac{1}{4}$  (as in the margin), they become

$\frac{13}{35}$	$\frac{7}{27}$	$\frac{8}{11}$	$\frac{1}{4}$
$\frac{91}{91}$	$\frac{245}{245}$	$\frac{245}{245}$	$\frac{27}{27}$
$\frac{26}{26}$	$\frac{11}{11}$	$\frac{70}{70}$	
$\frac{351}{351}$	$\frac{2695}{2695}$	$\frac{945 \times 11 = 10395}{945 \times 11 = 10395}$	$\frac{4}{4}$
$\frac{11}{11}$	$\frac{4}{4}$	$\frac{8}{8}$	$\frac{41580}{41580}$
$\frac{3861}{3861}$	$\frac{10780}{10780}$	$\frac{7560}{7560}$	$\frac{41580}{41580}$
$\frac{4}{4}$	$\frac{4}{4}$	$\frac{4}{4}$	$\frac{41580}{41580}$
$\frac{15444}{15444}$		$\frac{30240}{30240}$	

$$15444 + 10780 + 30240 + 10395 = 1\frac{25279}{41580}$$

$$\therefore 6\frac{13}{35} + 1\frac{7}{27} + \frac{8}{11} + \frac{1}{4} = 8\frac{25279}{41580}$$

(15) The fractions are  $\frac{5}{18} + \frac{7}{27} + \frac{19}{23} + 4\frac{1}{7}$ ; or, multiplying the terms of the last by 4, and adding to the preceding, they are

$$\frac{5}{18} + \frac{7}{27} + 4\frac{23}{28}$$

$$\frac{5}{2 \times 9} + \frac{7}{3 \times 9} = \frac{5 \times 3}{2 \times 9 \times 3} + \frac{7 \times 2}{3 \times 9 \times 2} = \frac{15 + 14}{54} = \frac{29}{54}.$$

$$\text{And } \frac{23}{28} + \frac{29}{54} = \frac{23}{14 \times 2} + \frac{29}{27 \times 2} = \frac{23 \times 27}{14 \times 2 \times 27} + \frac{29 \times 14}{27 \times 2 \times 14} = \frac{621 + 406}{756}$$

$$= \frac{1027}{756} = 1\frac{271}{756}. \therefore \frac{5}{18} + \frac{7}{27} + 4\frac{23}{28} = 5\frac{271}{756}.$$



(16) As no two of the denominators have a factor in common, we must proceed according to the rule with the fractions  $\frac{3}{13} + \frac{2}{13} + \frac{13}{11} + \frac{1}{5} + \frac{5}{7}$ . The work in the margin shows that the sum of these is

$$\frac{1155+910+1001+3575}{5005} = 1\frac{1636}{5005}, \text{ the sum required.}$$

$\begin{array}{r} 3 \\ 11 \\ \hline 33 \\ 5 \\ \hline 165 \\ 7 \\ \hline 1155 \end{array}$	$\begin{array}{r} 2 \\ 13 \\ \hline 26 \\ 5 \\ \hline 130 \\ 7 \\ \hline 910 \end{array}$	$\begin{array}{r} 13 \\ 11 \\ \hline 143 \\ 7 \\ \hline 1001 \\ 5 \\ \hline 3575 \end{array}$
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*Subtraction of Fractions, pages 76, 77.*

(1) Reducing the fractions  $\frac{4}{5}$ ,  $\frac{3}{4}$ , to a common denominator by the rule, they become  $\frac{16}{20}$ ,  $\frac{15}{20}$ ; so that  $3\frac{16}{20} - 2\frac{15}{20} = 1\frac{1}{20}$ .

(2) The fractions  $\frac{6}{7}$ ,  $\frac{5}{8}$ , in a common denominator are  $\frac{48}{56}$ ,  $\frac{35}{56}$ ;  $\therefore 1\frac{48}{56} - \frac{35}{56} = 1\frac{13}{56}$ .

(3) Reducing to a common denominator,  $\frac{11}{12} - \frac{3}{7} = \frac{77-36}{84} = \frac{41}{84}$ .

(4) Reducing to a common denominator,  $\frac{15}{16} - \frac{5}{8} = \frac{15-10}{16} = \frac{5}{16}$ .

(5) Reducing the fractions to a common denominator, we have  $\frac{20}{35} - \frac{21}{35}$ ; and as  $\frac{21}{35}$  is greater than  $\frac{20}{35}$ , we must borrow 1, that is  $\frac{35}{35}$ , from the integer 2 prefixed to the first fraction; this 1 added to  $\frac{20}{35}$  makes it  $\frac{55}{35}$ ; so that  $2\frac{20}{35} - 1\frac{21}{35} = 1\frac{55-21}{35} = 1\frac{34}{35}$ .

And whenever, as here, the subtractive fraction is the greater, we may subtract its numerator from the common denominator, add the remainder to the numerator of the other fraction, and put the denominator underneath: this being equivalent to increasing the smaller fraction by 1, we must take care to diminish the integer prefixed to the smaller fraction by 1. Should there be no prefixed integer, we shall know that the subtractive quantity exceeds the other, and therefore cannot be taken from it.

(6)  $\frac{7}{10} - \frac{10}{11} = \frac{77}{110} - \frac{100}{110}$ . Here, as the subtractive fraction is the greater, we subtract the 100 from the denominator 110, and adding the remainder 10 to the numerator 77, as directed above, we have  $4\frac{7}{10} - 3\frac{10}{11} = \frac{87}{110}$ .

A mere glance at the original fractions will in general enable us to see which of the two is the greater, as in the present instance, where we might have written down at once

$$1\frac{7}{10} - \frac{10}{11} = 1\frac{77}{110} - \frac{100}{110} = \frac{87}{110}.$$

(7) Here it is easy to see that the second fraction is the greater, therefore the example may be written

$$1\frac{13}{24} - \frac{17}{18} = 1\frac{39}{72} - \frac{68}{72} = \frac{43}{72}.*$$

(8) Here the subtractive 4 cancels the other; so that the example is the same as

$$\frac{19}{20} - \frac{15}{16} = \frac{76}{80} - \frac{75}{80} = \frac{1}{80}.$$

$$(9) \quad 1\frac{17}{32} - \frac{19}{24} = 1\frac{51}{96} - \frac{76}{96} = \frac{71}{96}.$$

$$(10) \quad \frac{13}{15} + \frac{9}{11} = \frac{143+135}{165} = \frac{278}{165} = 1\frac{113}{165}.$$

Again bringing  $\frac{113}{165}$ ,  $\frac{5}{12}$ , to a common denominator, by multiplying the terms of the first by 4, and those of the second by 55, we have  $1\frac{452}{660} - \frac{275}{660} = 1\frac{177}{660} = 1\frac{59}{220}$ .

Hence  $2 + 3 + 1\frac{59}{220} - 4 = 2\frac{59}{220}$ , the answer.

(11) This example is  $6\frac{7}{11} + 1\frac{3}{11} - 5\frac{8}{9}$ ; that is,  $7\frac{10}{11} - 5\frac{8}{9} = 2\frac{10}{11} - \frac{8}{9} = 2\frac{90}{99} - \frac{88}{99} = 2\frac{2}{99}$ .

(12) Here the integers cancel, for  $5 - 6 + 1 = 0$ ; therefore the Example is

$$\frac{7}{3} - \frac{3}{4} + \frac{5}{6} = \frac{14}{6} - \frac{3}{4} + \frac{5}{6} = \frac{19}{6} - \frac{3}{4} = \frac{38}{12} - \frac{9}{12} = \frac{29}{12} = 2\frac{5}{12}$$

\*  $72 - 68 = 4$ , and  $39 + 4 = 43$ . See the remarks appended to the work of Example 5.

(13) Since  $8 - 3 - 2 = 3$ , the Example is

$$3\frac{2}{3} - \frac{9}{16} - \frac{7}{15} = 3\frac{10}{15} - \frac{9}{16} - \frac{7}{15} = 3\frac{1}{5} - \frac{9}{16} = 3\frac{16}{80} - \frac{45}{80} = 2\frac{51}{80}^*$$

(14) This Example may be written

$$\begin{aligned} \frac{2}{3} + \frac{4}{7} - \frac{2}{15} - 1 &= \frac{10}{15} + \frac{4}{7} - \frac{2}{15} - 1 = \frac{8}{15} + \frac{4}{7} - 1 = \frac{56+60}{105} - 1 \\ &= \frac{116}{105} - 1 = \frac{11}{105}. \end{aligned}$$

(15) Since  $10 - 7 - 2 = 1$ , the Example is

$$1\frac{14}{15} - \frac{8}{35} - \frac{5}{12} = 1\frac{98}{105} - \frac{24}{105} - \frac{5}{12} = 1\frac{74}{105} - \frac{5}{12} = 1\frac{296}{420} - \frac{175}{420} = 1\frac{121}{420}$$

(16) Here  $4 - 2 - 1 = 1$ , and

$$1\frac{18}{55} - \frac{7}{11} - \frac{4}{5} = 1\frac{18}{55} - \frac{35}{55} - \frac{44}{55} = 1\frac{18}{55} - 1\frac{24}{55} = -\frac{6}{55}.$$

(17) Here  $3 - 1 + 5 - 2 = 5$ , and

$$5\frac{5}{6} - \frac{4}{7} + \frac{8}{9} = 5\frac{15}{18} - \frac{4}{7} + \frac{16}{18} = 6\frac{13}{18} - \frac{4}{7} = 6\frac{91}{126} - \frac{72}{126} = 6\frac{19}{126}.$$

(18)  $9 - 7 - 1 = 1$ , and

$$1\frac{17}{18} - \frac{2}{3} - \frac{2}{5} + \frac{1}{2} = 1\frac{17}{18} - \frac{12}{18} - \frac{2}{5} + \frac{9}{18} = 1\frac{7}{9} - \frac{2}{5} = 1\frac{35}{45} - \frac{18}{45} = 1\frac{17}{45}$$

(19)  $11 - 6 - 3 - 1 = 1$ , and

$$\begin{aligned} 1\frac{13}{16} - \frac{7}{12} - \frac{1}{3} - \frac{1}{5} &= 1\frac{13}{16} - \frac{7}{12} - \frac{4}{12} - \frac{1}{5} = 1\frac{13}{16} - \frac{11}{12} - \frac{1}{5} \\ &= 1\frac{39}{48} - \frac{44}{48} - \frac{1}{5} = \frac{43}{48} - \frac{1}{5} = \frac{215-48}{240} = \frac{167}{240} \end{aligned}$$

\*  $80 - 45 = 35$  and  $35 + 16 = 51$ . See remarks appended to the work of Ex. 5.

† It has not been thought necessary, in the work of these Examples, to exhibit at length the operations for reducing fractions to a common denominator; the learner ought not to require any aid in such reductions here. By looking at the denominators 105 and 12 above, it is readily seen that of the factors 3 and 4, of which 12 is composed, the former will divide 105, giving for quotient 35, thus showing that 105 may be replaced by  $35 \times 3$ , and 12 by  $4 \times 3$ ; so that the two become alike by multiplying the former by 4, and the latter by 35. The learner is expected to discover the common factors in this way for himself; he will often find a reference to the Table of Factors, at page 179 of the Arithmetic, useful in this business, though it gives no information about factors greater than 12.

(20)  $7 - 3 = 4$ , and

$$\frac{2}{5} + \frac{9}{11} - \frac{7}{18} - \frac{3}{10} = \frac{4}{10} + \frac{9}{11} - \frac{7}{18} - \frac{3}{10} = \frac{1}{10} + \frac{9}{11} - \frac{7}{18} = \frac{9}{90} + \frac{9}{11} - \frac{7}{18}$$

$$= \frac{35}{90} = \frac{9}{11} - \frac{26}{90} = \frac{810 - 286}{990} = \frac{524}{990} = \frac{262}{495}$$

 $\therefore$  adding the 4 above, the Ans. is  $4\frac{262}{495}$ (21)  $9 - 2 + 3 = 10$ , and

$$\frac{1}{3} - \frac{7}{15} + \frac{5}{18} + \frac{1}{2} = \frac{5}{15} - \frac{7}{15} + \frac{5}{18} + \frac{9}{18} = \frac{7}{9} - \frac{2}{15} = \frac{35 - 6}{45} = \frac{29}{45}$$

 $\therefore 10\frac{29}{45}$  is the Ans.(22)  $7 - 5 = 2$ , and  $\frac{1}{4} - \frac{1}{2} - \frac{7}{24} - \frac{15}{16} = \frac{12 - 24 - 14 - 45}{48} = -\frac{71}{48}$ 

$$\text{and } 2 - \frac{71}{48} = 1 - \frac{23}{48} = \frac{25}{48}$$

(23)  $13 - 12 = 1$ , and  $\frac{6}{11} - \frac{2}{11} - \frac{3}{8} - \frac{1}{12} = \frac{4}{11} - \frac{9}{24} - \frac{2}{24} = \frac{4}{11}$ 

$$- \frac{11}{24} = \frac{96 - 121}{264} = -\frac{25}{264}, \text{ and } 1 - \frac{25}{264} = \frac{239}{264}$$

(24)  $15 + 1 - 14 = 2$ , and  $\frac{7}{9} + \frac{1}{2} - \frac{11}{12} + \frac{3}{5} = \frac{7}{9} + \frac{6}{12} - \frac{11}{12} +$ 

$$\frac{3}{5} = \frac{7}{9} - \frac{5}{12} + \frac{3}{5} = \frac{28 - 15}{36} + \frac{3}{5} = \frac{13}{36} + \frac{3}{5} =$$

$$\frac{65 + 108}{180} = \frac{173}{180}, \text{ and adding the 2, the Answer}$$

$$\text{is } 2\frac{173}{180}.$$

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### *Multiplication of Fractions, page 80.*

$$(1) \quad * \frac{\frac{4}{15}}{6} \times \frac{7}{24} = \frac{1 \times 7}{15 \times 6} = \frac{7}{90}.$$

\* The numbers placed above the numerators and below the denominators, are the factors which remain after those common to both are cancelled. Thus, in Example 2, the common factor 2 is cancelled in the first numerator and the second denominator, the remaining factor 12 of the numerator being placed above, and the remaining factor 11 of the denominator below; which remaining factors are to replace the numbers themselves in the multiplication. In like manner in Example 3, the first and last numerators being the factors of the second denominator, all three are expunged, and the remaining factor of each, namely, 1, used instead; also since the third numerator cancels the factor 2 in the last denominator, the numerator and denominator become replaced by 1 and 4.

$$(2) \frac{\frac{12}{24}}{7} \times \frac{65}{\frac{22}{11}} = \frac{12 \times 65}{7 \times 11} = \frac{780}{77} = 10\frac{10}{77}.$$

$$(3) \frac{\frac{1}{3}}{\frac{4}{4}} \times \frac{7}{\frac{9}{1}} \times \frac{\frac{1}{2}}{\frac{5}{5}} \times \frac{\frac{1}{3}}{\frac{8}{4}} = \frac{7}{80}.$$

$$(4) \frac{\frac{1}{3}}{\frac{1}{1}} \times \frac{5}{7} \times \frac{\frac{2}{6}}{11} = \frac{10}{77}.$$

$$(5) \frac{\frac{1}{2}}{\frac{9}{9}} \times \frac{79}{\frac{16}{4}} \times \frac{\frac{1}{2}}{\frac{3}{3}} \times \frac{5}{1} = \frac{395}{108} = 3\frac{71}{108}.$$

$$(6) \frac{\frac{1}{8}}{\frac{5}{1}} \times \frac{18}{7} \times \frac{\frac{5}{25}}{\frac{8}{1}} = \frac{90}{7} = 12\frac{6}{7}.$$

$$(7) \frac{\frac{7}{35}}{\frac{12}{12}} \times \frac{50}{\frac{10}{2}} \times \frac{7}{1} = \frac{2801}{24} = 120\frac{11}{24}.$$

$$(8) \frac{601}{9} \times \frac{13}{\frac{14}{1}} \times \frac{\frac{2}{28}}{5} = \frac{15626}{45} = 347\frac{11}{45}.$$

$$(9) \frac{23}{5} \times \frac{\frac{2}{6}}{7} \times \frac{11}{\frac{3}{1}} = \frac{506}{35} = 14\frac{10}{35}.$$

$$(10) \frac{\frac{1}{2}}{\frac{7}{1}} \times \frac{30}{4} \times \frac{1}{5} \times \frac{\frac{1}{7}}{\frac{2}{1}} = \frac{39}{20} = 1\frac{19}{20}.$$

$$(11) \frac{\frac{1}{2}}{\frac{3}{3}} \times \frac{25}{\frac{6}{3}} \times \frac{7}{9} \times \frac{5}{1} = \frac{875}{81} = 10\frac{65}{81}.$$

$$(12) \frac{5}{\frac{6}{3}} \times \frac{50}{8} \times \frac{5}{3} \times \frac{\frac{1}{2}}{7} = \frac{1475}{504} = 2\frac{467}{504}.$$

$$(13) \frac{23}{\frac{7}{1}} \times \frac{21}{5} \times \frac{7}{8} \times \frac{1}{\frac{7}{1}} = \frac{3361}{40} = 84\frac{21}{40}.$$

$$(14) \frac{\frac{35}{11} \times \frac{1}{3} \times \frac{5}{2} \times \frac{7}{4}}{1} = \frac{1225}{6} = 204\frac{1}{6}.$$

$$(15) \frac{\frac{4}{3} \times \frac{1}{3} \times \frac{1}{7} \times \frac{8}{9}}{1} = \frac{32}{7} = 4\frac{4}{7}.$$

$$(16) \frac{\frac{107}{5} \times \frac{3}{2} \times \frac{1}{9} \times \frac{2}{45}}{1} = \frac{214}{675}.$$



*Division of Fractions, page 81.*

(1) Expunging the factor 7, common to the denominators, the example is

$$\frac{3}{1} \div \frac{11}{4}, \text{ and } \frac{3}{1} \times \frac{4}{11} = \frac{12}{11} = 1\frac{1}{11}.$$

(2) Expunging the factor 2 from the numerators, and 3 from the denominators, we have

$$\frac{7}{5} \times \frac{9}{8} = \frac{63}{40} = 1\frac{23}{40}.$$

$$(3) \quad 2\frac{13}{14} \div 4\frac{7}{8} = \frac{41}{14} \div \frac{39}{8} = \frac{41}{7} \div \frac{39}{4} = \frac{41}{7} \times \frac{4}{39} = \frac{164}{273}.$$

$$(4) \quad 3\frac{12}{35} \div 7\frac{2}{15} = \frac{117}{35} \div \frac{107}{15} = \frac{117}{7} \div \frac{107}{3} = \frac{117}{7} \times \frac{3}{107} = \frac{351}{740}.$$

$$(5) \quad 1\frac{7}{10} \div 10\frac{1}{5} = \frac{17}{10} \div \frac{51}{5} = \frac{1}{2} \div \frac{3}{1} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.$$

$$(6) \quad \frac{27}{1} \div \frac{8}{15} = \frac{27}{1} \times \frac{15}{8} = \frac{405}{8} = 50\frac{5}{8}.$$

$$(7) \quad \frac{3}{5} \times \frac{2}{7} \div \frac{5}{8} = \frac{3}{5} \times \frac{2}{7} \times \frac{8}{5} = \frac{48}{175}.$$

$$(8) \quad \frac{2}{3} \times \frac{13}{9} \div \frac{40}{11} = \frac{1}{3} \times \frac{13}{9} \times \frac{11}{20} = \frac{143}{540}.$$

$$(9) \quad \frac{3}{7} \times \frac{5}{2} \div \frac{1}{2} \times \frac{11}{4} = \frac{3}{7} \times \frac{5}{1} \div \frac{1}{1} \times \frac{11}{4} = \frac{15}{7} \div \frac{1}{4} = \frac{15}{7} \times \frac{4}{11} = \frac{60}{77}.$$

$$(10) \quad \frac{41}{9} \div \frac{3}{8} \times \frac{1}{4} \times \frac{3}{5} = \frac{41}{9} \div \frac{9}{5} = \frac{41}{9} \times \frac{5}{9} = \frac{205}{81} = 2\frac{43}{81}.$$

$$(11) \quad \frac{18}{1} \div \frac{6}{11} \times \frac{2}{3} \times \frac{1}{4} = \frac{3}{1} \div \frac{1}{11} \times \frac{2}{3} \times \frac{1}{4} = \frac{3}{1} \div \frac{1}{66} = 3 \times 66 = 198.$$

$$(12) \quad \text{First: } 3 \div 7\frac{3}{5} = 3 \div \frac{38}{5} = \frac{3}{1} \times \frac{5}{38} = \frac{15}{38}.$$

$$\text{Second: } 2\frac{1}{4} \div 3\frac{2}{3} = \frac{9}{4} \div \frac{11}{3} = \frac{9}{4} \times \frac{3}{11} = \frac{27}{44}.$$

$$\text{Then } \frac{15}{38} \div \frac{27}{44} = \frac{5}{19} \div \frac{9}{22} = \frac{5}{19} \times \frac{22}{9} = \frac{110}{171}.$$

The preceding example may be worked rather differently, as follows:—

If the numerator and denominator of the first complex fraction be multiplied by 5, the complex fraction will be replaced by the simple fraction  $\frac{15}{38}$ ; and if the numerator and denominator of the second complex fraction be multiplied by 12, it will become the simple fraction  $\frac{27}{44}$ : therefore the example is the same as  $\frac{15}{38} \div \frac{27}{44}$ .



### *Reduction of One Quantity to a Fraction of Another,* page 83.

Multiplying num. and den. by 12, we have

$$(1) \quad \frac{110s.}{5s.} = \frac{22}{60} = \frac{11}{30} \text{ so that } 1s. 10d. \text{ is the } \frac{11}{30} \text{th of } 5s.$$

$$(2) \quad \frac{8\frac{1}{2}d.}{30d.} = \frac{17}{60} \therefore \frac{17}{60} \text{ of } 2s. 6d. = 8\frac{1}{2}d.$$

$$(3) \quad \frac{3\frac{1}{2}s.}{21s.} = \frac{47}{252} \therefore 3s. 11d. = \frac{47}{252} \text{ of } 21s.$$

$$(4) \quad \frac{3}{7} \text{ of } 2s. = \frac{6}{7}s., \text{ and } \frac{\frac{6}{7}s.}{20s.} = \frac{6}{140} = \frac{3}{70} \therefore \frac{3}{7} \text{ of } 2s. \text{ is the same as } \frac{3}{70} \text{ of } £1.$$

$$(5) \frac{2\frac{5}{12} \text{ ft.}}{3 \text{ ft.}} = \frac{29}{36} \therefore 2 \text{ ft. } 5 \text{ in. is } \frac{29}{36} \text{ of a yard.}$$

$$(6) \frac{3\frac{1}{4} \text{ cwt.}}{20 \text{ cwt.}} = \frac{13}{80} \therefore 3 \text{ cwt. } 1 \text{ qr. is } \frac{13}{80} \text{ of 1 ton.}$$

$$(7) \frac{9}{11} \text{ of } 10 \text{ min.} = \frac{90}{11} \text{ min., and } \frac{\frac{90}{11} \text{ min.}}{60 \text{ min.}} = \frac{90}{660} = \frac{3}{22} \therefore \frac{9}{11} \text{ of } 10 \text{ min. is } \frac{3}{22} \text{ of 1 hour.}$$

$$(8) \frac{1}{3} \text{ of } 3\frac{2}{3} = \frac{1}{3} \times \frac{23}{7} = \frac{23}{21} \text{ and } \frac{\frac{23}{21}}{\frac{23}{21}} = \frac{23}{21} \div \frac{13}{5} = \frac{23}{21} \times \frac{5}{13} \\ = \frac{115}{273} \therefore \frac{1}{3} \text{ of } 3\frac{2}{3} \text{ is } \frac{115}{273} \text{ of } 2\frac{1}{3}.$$

$$(9) \frac{7\frac{1}{2} \text{ s.}}{20 \times 1\frac{1}{2} \text{ s.}} = \frac{37}{5 \times 20 \times 1\frac{1}{2}} = \frac{37 \times 7}{5 \times 20 \times 10} = \frac{259}{1000}, \text{ so that } 7\frac{1}{2} \text{ s.} \\ \text{is } \frac{259}{1000} \text{ of } £1\frac{1}{2}.$$

$$(10) 12^\circ 23\frac{1}{2}' = 743\frac{1}{2}'; 90^\circ = 5400'; \text{ and } \frac{743\frac{1}{2}'}{5400'} = \frac{1487}{10800} \\ \therefore 12^\circ 23\frac{1}{2}' \text{ is } \frac{1487}{10800} \text{ of } 90^\circ.$$

$$(11) \frac{21\frac{1}{4} \text{ d.}}{7 \text{ d.}} = \frac{65}{168} \therefore 2 \text{ d. } 17 \text{ h. is } \frac{65}{168} \text{ of a week.}$$

$$(12) \frac{7}{15} \text{ of } 2\frac{1}{3} = \frac{7}{15} \times \frac{5}{2} = \frac{7}{6} \text{ and } \frac{7}{6} \div \frac{3}{2} = \frac{7}{6} \times \frac{2}{3} = \frac{7}{9} \\ \therefore \frac{7}{15} \text{ of } 2\frac{1}{3} \text{ is } \frac{7}{9} \text{ of } 1\frac{1}{3}.$$

$$(13) \frac{7}{15} \text{ of } 1\frac{1}{4} = \frac{7}{15} \times \frac{5}{4} = \frac{49}{60} \text{ and } \frac{49}{60} \div \frac{5}{2} = \frac{49}{30} \div \frac{5}{1} = \frac{49}{30} \\ \times \frac{1}{5} = \frac{49}{150} \therefore \frac{7}{15} \text{ of } 1\frac{1}{4} \text{ is } \frac{49}{150} \text{ of } 2\frac{1}{2}.$$



(14)

£	s.	d.	£	s.	d.
7	13	4½	2	14	2½
<u>20</u>			<u>20</u>		
153			54		
<u>12</u>			<u>12</u>		
1840			650		
<u>4</u>			<u>4</u>		
7362			2601		

∴ the fraction is  $\frac{2601}{7362}$ , or dividing each of its terms by 9, it is  $\frac{289}{818}$ .

(15)

roo.	po.	yd.	ac.	po.	yd.
3	21	3	11	2	6
<u>40</u>			<u>4</u>		
4)141			46		
<u>30½</u>			<u>40</u>		
4233			4)1846		
<u>35½</u>			<u>30½</u>		
4268½ yd.			55380		
			<u>461½</u>		
			55841½ yd.		

∴ the fraction is

$$\frac{4268\frac{1}{2}}{55841\frac{1}{2}} = \frac{17073}{223366}$$

*Multiplication and Division of Concrete Quantities, p. 85.*

(1)

£	s.	d.	
12	11	6	
		<u>12½</u>	
150	18	0	
for ½	6	5	9
for ¼	3	2	10½
£160	6	7½	

(2)

$£2\frac{3}{11} \times \frac{2}{3} = £\frac{4}{11} \times \frac{2}{3} = £\frac{8}{33}$ , the value in fraction of a £.

75	
<u>20</u>	
77)1500(19s. 5½d. + ⅔f.	
<u>77</u>	
730	
<u>693</u>	
37	
<u>12</u>	
444	
<u>385</u>	
59	
<u>4</u>	
236	
<u>231</u>	
5	

(3)

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 17 \quad 5\frac{1}{2} \\ \hline 12 \\ \hline 209 \quad 3019 \\ 4 \quad 837 \\ \hline 837 \text{ f.} \quad 21133 \\ 9057 \\ \hline 24152 \\ 4) 2526903 \\ 4) 631725\frac{3}{4} \text{ f.} \\ 12) 157931\frac{1}{2} \text{ d.} \\ 2,0) 1316,0 \text{ s. } 11 \text{ d.} \\ \hline \pounds 658 \text{ 0s. } 11\frac{1}{2} \text{ d.} + \frac{3}{4} \text{ f.} \end{array}$$

Otherwise :

$$\begin{array}{r} 837 = 847 - 10 = 11 \times 11 \times 7 \\ - 10 \text{ (See Table, p. 179.)} \\ 754\frac{3}{4} \times 10 \\ \hline 11 \\ \hline 8302\frac{3}{4} \\ 11 \\ \hline 91324\frac{3}{4} \\ 7 \\ \hline 639273\frac{1}{2} \\ \text{Subtract } 7547\frac{3}{4} \\ \hline 4) 631725\frac{3}{4} \text{ f.} \\ 12) 157931\frac{1}{2} \text{ d.} \\ 2,0) 1316,0 \text{ 11} \\ \hline \pounds 658 \text{ 0s. } 11\frac{1}{2} \text{ d.} + \frac{3}{4} \text{ f.} \end{array}$$

$$(4) \quad 77\frac{1}{2} \div 10\frac{1}{2} = \frac{387}{5} \times \frac{2}{21} = \frac{77\frac{1}{2}}{105} \text{ miles.}$$

$$105) 774 (7 \text{ m. } 2\frac{1}{3} \text{ fur.}$$

$$\begin{array}{r} 735 \\ \hline 39 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 312 \\ \hline 210 \\ \hline 102 \end{array}$$

$$\begin{array}{r} 102 \\ \hline 102 \end{array}$$

$$\begin{array}{r} 312 \\ \hline 210 \\ \hline 102 \end{array}$$

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$$\begin{array}{r} 102 \\ \hline 102 \end{array}$$

(5)

$$\begin{array}{r} 11\frac{3}{5} \\ 7 \end{array}$$

$$\begin{array}{r} 11\frac{3}{5} \\ 7 \end{array}$$

$$\begin{array}{r} 11\frac{3}{5} \\ 7 \end{array}$$

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$$\begin{array}{r} 11\frac{3}{5} \\ 7 \end{array}$$

$$\begin{array}{r} 11\frac{3}{5} \\ 7 \end{array}$$

 $81\frac{1}{2}$  No. of yards.

$$\pounds 54\frac{3}{5} \div 81\frac{1}{2} = \pounds 8\frac{408}{5} \div \frac{163}{2} = \pounds 8\frac{408}{5} \times \frac{2}{163} = \pounds \frac{8176}{812} * = \frac{21730}{1624} \text{ s.} = \frac{10875}{812} \text{ s.}$$

$$812) 10875 (13 \text{ s. } 4\frac{1}{2} \text{ d.} + \frac{5}{8} \text{ f.}$$

$$\begin{array}{r} 812 \\ \hline 2755 \\ \hline 2436 \\ \hline 319 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 3828 \\ \hline 3248 \\ \hline 580 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 2320 \\ \hline 1624 \\ \hline 696 \end{array}$$

$$\begin{array}{r} 696 \\ \hline 696 \end{array}$$

$$\begin{array}{r} 696 \\ \hline 696 \end{array}$$

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$$\begin{array}{r} 696 \\ \hline 696 \end{array}$$

$$\begin{array}{r} 696 \\ \hline 696 \end{array}$$

\* To reduce this to shillings, the numerator may be multiplied by 20; but as it is obvious that the denominator will divide by 2, the numerator may be multiplied by 10 only, and half the denominator taken. (See p. 82, Arithmetic.)

$$(6) \begin{array}{r} \text{£} \\ 625 \\ \underline{2} \end{array}$$

$$7)1250$$

$$A, \text{£}178 \text{ 11s. } 5\frac{1}{2}d.$$

$$\begin{array}{r} \text{£} \\ 625 \\ \underline{2} \end{array}$$

$$9)1250 \text{ s. d.}$$

$$B, \text{£}138 \text{ 17 } 9\frac{1}{2}$$

$$178 \text{ 11 } 5\frac{1}{2}$$

$$\text{Subtract from £625, to get C, } \begin{array}{r} \text{£}317 \text{ 9 } 2\frac{1}{2} \\ \underline{\text{£}307 \text{ 10 } 9\frac{1}{2}} \end{array} \left\{ \begin{array}{l} \text{sum of shares of A} \\ \text{and B.} \end{array} \right.$$

$$(7) \frac{9}{10} \text{ of } 6884875$$

$$\underline{9}$$

$$4)6196387,5 \text{ English dollars.}$$

$$\text{£}1549096 \text{ } 3\frac{5}{10} \text{ dol.}$$

And since 3 English dollars is 15s., and  $\frac{5}{10}$  dol., or  $\frac{1}{2}$  dol. = 2s. 6d., the amount, in English money, is £1549096 17s. 6d.

$$(8) \frac{2}{3} \text{ of } \frac{2}{3} \text{ of £900 is } \frac{1}{3} \text{ of } \frac{2}{3} \text{ of £900.}$$

$$\begin{array}{r} \text{£} \\ 3)900 \end{array}$$

$$4)300 = \frac{1}{4}$$

$$\frac{75}{5} = \frac{1}{5} \text{ of } \frac{1}{4}$$

$$\frac{2}{3} \text{ of } \frac{1}{3}^* = \text{£}375, \text{ the worth.}$$

$$\text{Or, since } \frac{1}{3} \text{ of } \frac{2}{3} = \frac{2}{9},$$

$$\begin{array}{r} \text{£} \\ 12)900 \end{array}$$

$$\frac{75}{5} = \frac{1}{12}$$

$$\text{£}375 = \frac{1}{12}.$$

(9) Here we have first to find what fraction of £864 10s £160 is; that is, we must reduce  $\frac{\text{£}160}{\text{£}864 \text{ 10s.}}$  to an ordinary fraction.

$$\text{This may be written } \frac{160}{864\frac{1}{2}} = \frac{320}{1729} = \text{the share of the property}$$

$$\text{disposed of } \therefore \text{ the share left is } \frac{5}{9} - \frac{320}{1729} = \frac{8645 - 2880}{15561} = \frac{5765}{15561}.$$

\* The learner can scarcely require to be reminded here that  $\frac{2}{3}$  of  $\frac{1}{3}$  is the same as  $\frac{1}{3}$  of  $\frac{2}{3}$ . One fraction of another is the product of the two; so that whether the first be multiplied by the second, or the second by the first, is matter of indifference.



*Greatest Common Measure, pages 88, 89.*

$$\begin{array}{r}
 (1) \quad 247 \overline{) 323} \quad (1 \\
 \underline{247} \\
 76 \overline{) 247} \quad (3 \\
 \underline{228} \\
 19
 \end{array}$$

Here 19 has no divisor except unit: consequently if a common measure exist it must be 19.

Trying 19 upon the preceding divisor 76, it is found to succeed.  $\therefore$  19 is the G. C. M.—(See Note, p. 86 of the Arithmetic.)

$$\begin{array}{r}
 (2) \quad 272 \overline{) 425} \quad (1 \\
 \underline{272} \\
 153 \overline{) 272} \quad (1 \\
 \underline{153} \\
 119 \overline{) 153} \quad (1 \\
 \underline{119} \\
 34
 \end{array}$$

Here the only divisors of 34 are 2 and 17, the latter of which alone divides 119  $\therefore$  17 is the G. C. M.

$$\begin{array}{r}
 (3) \quad 57 \overline{) 63} \quad (1 \\
 \underline{57} \\
 6
 \end{array}$$

The only divisors of 6 are 2 and 3, only the latter of which divides 57  $\therefore$  3 is the G. C. M.

$$\begin{array}{r}
 (4) \quad 408 \overline{) 527} \quad (1 \\
 \underline{408} \\
 119 \overline{) 408} \quad (3 \\
 \underline{357} \\
 51
 \end{array}$$

The only divisors of 51 are 3 and 17: the latter alone divides 119  $\therefore$  17 is the G. C. M.

$$\begin{array}{r}
 (5) \quad 1164 \overline{) 1261} \quad (1 \\
 \underline{1164} \\
 97
 \end{array}$$

97 has no divisor except unit, and it is found to divide 1164  $\therefore$  97 is the G. C. M.

$$\begin{array}{r}
 (6) \quad 297 \overline{) 1021} \quad (3 \\
 \underline{891} \\
 130
 \end{array}$$

The only divisors of 130 are 2, 5, and 13; and a glance at the last figure of 297 shows that it is useless to try 2 or 5, and on trying

13 it is found to fail  $\therefore$  the numbers have no common measure, so that the fraction is already in its lowest terms.

# 58 KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

$$(7) \quad 618 \overline{) 816} (1$$

$$\underline{618}$$

$$198 \overline{) 618} (3$$

$$\underline{594}$$

$$24 \overline{) 198} (8$$

$$\underline{192}$$

$$\underline{6}$$

6 divides 24  $\therefore$  6 is the G. C. M.; and dividing both terms of  $\frac{816}{618}$  by 6 there results  $\frac{136}{103}$ , which is the proposed fraction in its lowest terms.

$$(8) \quad 7163 \overline{) 15863} (2$$

$$\underline{14326}$$

$$1537 \overline{) 7163} (4$$

$$\underline{6148}$$

$$1015 \overline{) 1537} (1$$

$$\underline{1015}$$

$$522 \overline{) 1015} (1$$

$$\underline{522}$$

$$493 \overline{) 522} (1$$

$$\underline{493}$$

$\therefore$  the G. C. M. is 29; and dividing the terms of the fraction by this we have  $\frac{7163}{15863} = \frac{247}{547}$ .

$$29 \overline{) 493} (17$$

$$\underline{29}$$

$$\underline{203}$$

$$\underline{203}$$

$$(9) \quad 5283 \overline{) 176491} (33$$

$$\underline{15849}$$

$$\underline{18001}$$

$$\underline{15849}$$

$$2152 \overline{) 5283} (2$$

$$\underline{4304}$$

$$979 \overline{) 2152} (2$$

$$\underline{1958}$$

$$194 \overline{) 979} (5$$

$$\underline{970}$$

$$\underline{9}$$

$\therefore$  since 3, the only divisor of 9, fails to divide 194, there is no C. M.

$$(10) \quad 156 \overline{) 169} (1$$

$$\underline{156}$$

$$13 \overline{) 156} (12$$

$$\underline{13}$$

$$\underline{26}$$

$$\underline{26}$$

$\therefore$  13 rods is the required length.

$$\begin{array}{r} (11) \quad 323)456(1 \\ \underline{323} \\ 133)323(2 \\ \underline{266} \end{array}$$

$$\begin{array}{r} 57)133(2 \\ \underline{114} \\ 19)57(3 \\ \underline{57} \end{array}$$

$\therefore$  the G. C. M. of 323 and 456 is 19.

$$\begin{array}{r} \text{Again :} \quad 19)551(29 \\ \underline{38} \\ 171 \\ \underline{171} \end{array}$$

$\therefore$  19 is the G. C. M. of 323, 456, and 551  $\therefore$  £19 is the price per acre: and  $\frac{£323}{19} =$

$$£17, \quad \frac{£456}{19} = £24, \quad \frac{£551}{19} = £29.$$

(12) See foot-note p. 89, Arithmetic.

$$\begin{array}{r} \begin{array}{ccc} s. & s. & d. \\ 10 & 7 & 10\frac{1}{2} \\ \underline{12} & \underline{12} & \\ 120 & 94 & \\ \underline{2} & \underline{2} & \\ 240 & 189 & \end{array} \left( \frac{189}{240} = \frac{63}{80} \right. \end{array}$$

Hence the least number by which the dividend can be multiplied to prevent a fraction in the quotient is 80, which must therefore be the least number of ounces. The quotient after this multiplication of the dividend by 80, will

be 63; and as £3 10s. has been omitted in the dividend, seven half-sovereigns have been omitted  $\therefore$  the entire number of half-sovereigns is  $80 \times 7 + 63 = 623$ .

*Least Common Multiple, page 91.*

$$\begin{array}{r} (1) \quad 8)12(1\frac{1}{2} \\ \underline{2} \\ 18)24(1\frac{1}{3} \\ \underline{3} \end{array}$$

$$\begin{array}{r} 20)72(3\frac{3}{5} \\ \underline{5} \end{array}$$

360, the L. C. M.

(2) Here the second contains the first, the third the second, and the fourth the third; so that 81 is the L. C. M.

$$(3) \quad 6)8(1\frac{1}{3} \\ \underline{3}$$

$$9)24(2\frac{2}{3} \\ \underline{3}$$

$$\underline{72} \times 5 \times 7 = 2520, \text{ the L. C. M.}$$

$$(4) \quad 16)24(1\frac{1}{2} \\ \underline{2}$$

$$20)48(2\frac{1}{5} \\ \underline{5}$$

$$\begin{array}{r} 30)240(8 \\ 25)240(9\frac{3}{5} \\ \underline{5} \end{array}$$

1200, the L. C. M.

$$(5) \quad \begin{array}{r} 24)27(1\frac{1}{3} \\ \underline{8} \end{array}$$

$$\begin{array}{r} 15)216(14\frac{2}{3} \\ \underline{5} \end{array}$$

$$\begin{array}{r} 126)1080(8\frac{1}{2} \\ \underline{7} \end{array}$$

7560, the L. C. M.

$$(6) \quad \begin{array}{r} 21)427(20\frac{1}{3} \\ \underline{3} \end{array} \quad \begin{array}{r} 242)748(3\frac{1}{11} \\ \underline{11} \end{array}$$

$$\begin{array}{r} 1281 \\ \hline \end{array}$$

)

$$\begin{array}{r} 8228(6 \\ \underline{7686} \end{array}$$

$$\begin{array}{r} 542)1281(2 \\ \underline{1084} \end{array}$$

As 49 has no divisor but 7, which is not contained in 148, the numbers 1281, 8228, have no common measure  $\therefore$  the L. C. M. of the proposed numbers is

$$8228 \times 1281 = 10540068^*.$$

$$\begin{array}{r} 197)542(2 \\ \underline{394} \end{array}$$

$$\begin{array}{r} 148)197(1 \\ \underline{148} \end{array}$$

$$\begin{array}{r} 49 \\ \hline \end{array}$$

The work of Example 6 exhibits a little departure from the rule: the L. C. M. of 21, 427 is first found to be 1281; the L. C. M. of 242, 748 is then found to be 8228; and as these results have no common measure, their product is the L. C. M. required.

(7) As the numbers 11, 13, 17, 19, are prime numbers, they may be reserved till the L. C. M. of the others is found.

$$\begin{array}{r} 12)14(1\frac{1}{3} \\ \underline{6} \end{array}$$

$$\begin{array}{r} 16)18(1\frac{1}{8} \\ \underline{8} \end{array}$$

$$\begin{array}{r} 19 \\ \hline \end{array}$$

$$\begin{array}{r} 15)84(5\frac{1}{3} \\ \underline{5} \end{array}$$

$$\begin{array}{r} 20)144(7\frac{1}{5} \\ \underline{5} \end{array}$$

$$\begin{array}{r} 133 \\ \hline \end{array}$$

$$\begin{array}{r} 21)420(20 \\ \underline{7} \end{array}$$

$$\begin{array}{r} 420)720(1\frac{2}{3} \\ \underline{7} \end{array}$$

$$\begin{array}{r} 323 \\ \hline \end{array}$$

$$\begin{array}{r} 5040 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ \hline \end{array}$$

$$\begin{array}{r} 969 \\ \hline \end{array}$$

$$\begin{array}{r} 323 \\ \hline \end{array}$$

$$\begin{array}{r} 4199 \\ \hline \end{array}$$

$$\begin{array}{r} 11 \\ \hline \end{array}$$

46189 L. C. M. of the primes.

5040 L. C. M. of the Comp. Nos.

$$1847560$$

$$\begin{array}{r} 230945 \\ \hline \end{array}$$

$$\begin{array}{r} 232792560 \\ \hline \end{array} \text{ L. C. M. of all.}$$

In the above work the L. C. M. of 12, 15, 21, is found to be 420; the L. C. M. of 16, 20, 420, is found to be 5040, and this multiplied by the product of the reserved prime numbers, gives 232792560 for the L. C. M. of the proposed numbers.

\* For the quotient of 8228 divided by 1281 is  $6\frac{442}{1281}$ , the fraction being in its lowest terms.



There was no necessity to take account of the divisor 20, in the foregoing operation, because 20 is obviously contained in the dividend 420 already found. If the 20 had been passed over, then 144 would have been taken for a divisor, and 420 for the corresponding dividend; the quotient would have been  $2\frac{32}{44} = 2\frac{1}{11}$ , so that we should have had  $420 \times 12 = 5040$ , as before.

It is well worthy of notice, that as, in the above method of finding the L. C. M., we make use only of the denominators of the fractions in the quotients, after reduction to the lowest terms, it is matter of indifference whether our complete quotient in any case be an integer *increased* by a fractional correction, or *decreased* by such correction, so that we may always, if we please, increase the integral part of the true quotient by a unit, and use the corresponding fractional correction without paying any regard as to whether it be additive or subtractive; \* as, for instance, in dividing 420 by 144, as above, we get a remainder inconveniently large, namely, 132; but, by taking the integral part of the quotient, a unit greater, namely, 3, the remainder, given by subtracting the upper number from the lower, is 12, and  $\therefore$  the fraction is  $\frac{12}{144}$  or  $\frac{1}{12}$ . In like manner, in dividing 84 by 15 above, the denominator we seek is more readily discovered by taking 6 times 15, than by taking 5 times 15, as in the operation; for in no case need the remainder exceed half the divisor.

There are, of course, several ways of varying the form of the operation for the L. C. M., as we may take the proposed numbers in any order we please, and may also separate the work into two or more sets of divisors, as above, whenever those numbers are numerous, in which case a little consideration as to the most eligible of all the ways suggested will be advisable. The work of the last Example, as in the margin, and which is in strict accordance with the rule, is somewhat preferable to the mode of proceeding above. The divisor 20 is suppressed because of the dividend 60.

$$\begin{array}{r}
 12)15(1\frac{1}{2} \\
 \underline{4} \\
 60 \\
 \underline{4} \\
 18)240(10\frac{10}{3} \\
 \underline{3} \\
 21)720(30\frac{30}{7} \\
 \underline{7} \\
 5040
 \end{array}$$

NOTE.—On a Method of Shortening the Operation for the G. C. M.  
 —The principle to which notice has been invited in the foregoing remarks may be applied with advantage to the process for the

\* The terms of a fraction will always have the same common measure as the terms of the fraction which results from subtracting the former from unit; for since the two fractions have a common denominator, and, together, make a unit; if one be reduced to lower terms, and then subtracted from unit, the remainder must be the other; and since this remainder has the same denominator as the fraction subtracted, the terms of each of the two original fractions must have been divided by the same number.

common measure ;—the principle, namely, of always securing the *smallest* remainder at each step of the work. I shall here exemplify its application in two or three Examples taken from other books, and the work of which, as given in those books, I shall place beside the modified form of operation here recommended :—

*Improved Way.*

$$\begin{array}{r} 4155)24720(6 \\ \underline{24930} \end{array}$$

$$\begin{array}{r} 210)4155(20 \\ \underline{4200} \end{array}$$

$$\begin{array}{r} 45)210(5 \\ \underline{225} \end{array}$$

$$\begin{array}{r} G. C. M. \quad 15)45(3 \\ \underline{45} \end{array}$$

*Common Way.*

$$\begin{array}{r} 4155)24720(5 \\ \underline{20775} \end{array}$$

$$\begin{array}{r} 3945)4155(1 \\ \underline{3945} \end{array}$$

$$\begin{array}{r} 210)3945(18 \\ \underline{3780} \end{array}$$

$$\begin{array}{r} 165)210(1 \\ \underline{165} \end{array}$$

$$\begin{array}{r} 45)165(3 \\ \underline{135} \end{array}$$

$$\begin{array}{r} 30)45(1 \\ \underline{30} \end{array}$$

$$\begin{array}{r} G. C. M. \quad 15)30(2 \\ \underline{30} \end{array}$$

*Common Way.*

$$\begin{array}{r} 3775)10000(2 \\ \underline{7550} \end{array}$$

$$\begin{array}{r} 2450)3775(1 \\ \underline{2450} \end{array}$$

$$\begin{array}{r} 1325)2450(1 \\ \underline{1325} \end{array}$$

$$\begin{array}{r} 1125)1325(1 \\ \underline{1125} \end{array}$$

$$\begin{array}{r} 200)1125(5 \\ \underline{1000} \end{array}$$

*Improved Way.*

$$\begin{array}{r} 3775)10000(3 \\ \underline{11325} \end{array}$$

$$\begin{array}{r} 1325)3775(3 \\ \underline{3975} \end{array}$$

$$\begin{array}{r} 200)1325(6 \\ \underline{1200} \end{array}$$

$$\begin{array}{r} G. C. M. \quad 25)200(8 \\ \underline{200} \end{array}$$

$$\begin{array}{r} 125)200(1 \\ \underline{125} \end{array}$$

$$\begin{array}{r} 75)125(1 \\ \underline{75} \end{array}$$

$$\begin{array}{r} 50)75(1 \\ \underline{50} \end{array}$$

$$\begin{array}{r} G. C. M. \quad 25)50(2 \\ \underline{50} \end{array}$$

*Common Way.*

$$\begin{array}{r}
 2314)3721(1 \\
 \underline{2314} \\
 1407)2314(1 \\
 \underline{1407} \\
 907)1407(1 \\
 \underline{907} \\
 500)907(1 \\
 \underline{500}
 \end{array}$$

*Improved Way.*

$  \begin{array}{r}  2314)3721(2 \\  \underline{4628} \\  907)2314(3 \\  \underline{2721} \\  407)907(2 \\  \underline{814} \\  93)407(4 \\  \underline{372} \\  35)93(3 \\  \underline{105} \\  12)35(3 \\  \underline{36} \\  G. C. M. \underline{1}  \end{array}  $	$  \begin{array}{r}  407)500(1 \\  \underline{407} \\  93)407(4 \\  \underline{372} \\  35)93(2 \\  \underline{70} \\  23)35(1 \\  \underline{23} \\  12)23(1 \\  \underline{12} \\  11)12(1 \\  \underline{11} \\  G. C. M. \underline{1}  \end{array}  $
--	--

*Practice, pages 96, 97.*

<p>(1) 37 at £2 16s. 6d. *</p> $  \begin{array}{r}  2 \\  \underline{2} \\  \text{of } £2, 8s. \left  \begin{array}{l} 74 \\ 14 \ 16 \end{array} \right. \\  \text{of } £2, 8s. \left  \begin{array}{l} 14 \ 16 \\ 18 \ 6 \end{array} \right. \\  37 \text{ sixpences} \\  \underline{\underline{£104 \ 10 \ 6}}  \end{array}  $	<p>(2) 41 at £3 17s. 8d.</p> $  \begin{array}{r}  3 \\  \underline{3} \\  \text{of } £3, 15s. \left  \begin{array}{l} 123 \\ 30 \ 15 \end{array} \right. \\  2s. 6d. \left  \begin{array}{l} 5 \ 2 \ 6 \\ 6 \ 10 \end{array} \right. \\  41 \text{ twopences} \\  \underline{\underline{£159 \ 4 \ 4}}  \end{array}  $
--	--

\* The learner will not forget that Practice is nothing but a short and convenient method for the multiplication of compound quantities. In the present Example it is required to multiply £2 16s. 6d. by 37; the

(3) 79 at £5 11s. 7d.

			5
			<hr/>
			395
of £1, 10s.	$\left  \begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right $	39	10
1s.	$\left  \begin{smallmatrix} 1 \\ 10 \end{smallmatrix} \right $	3	19
6d.	$\left  \begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right $	1	19 6
79 pence		6	7
			<hr/>
			£440 15 1

(4) 83 at £6 3s. 6½d.

			6
			<hr/>
			498
of £1, 2s.	$\left  \begin{smallmatrix} 1 \\ 10 \end{smallmatrix} \right $	8	6
1s.	$\left  \begin{smallmatrix} 1 \\ 10 \end{smallmatrix} \right $	4	3
6d.	$\left  \begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right $	2	1 6
83 halfpence		3	5½
			<hr/>
			£512 13 11½

(5) 133 at £1 13s. 8½d.

			10s.	$\left  \begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right $	66	10
			2s.	$\left  \begin{smallmatrix} 1 \\ 10 \end{smallmatrix} \right $	13	6
			1s.	$\left  \begin{smallmatrix} 1 \\ 10 \end{smallmatrix} \right $	6	13
of 2s., 8d.	$\left  \begin{smallmatrix} 1 \\ 3 \end{smallmatrix} \right $	4	8	8		
133 halfpence		5	6½			
						<hr/>
						£224 3 2½

(6) 211 at £3 9s. 5¼d.

			3
			<hr/>
			633
of £1 { 5s.	$\left  \begin{smallmatrix} 1 \\ 4 \end{smallmatrix} \right $	52	15
of £1 { 4s.	$\left  \begin{smallmatrix} 1 \\ 5 \end{smallmatrix} \right $	42	4
of 5s., 5d.	$\left  \begin{smallmatrix} 1 \\ 12 \end{smallmatrix} \right $	4	7 11
½d.	$\left  \begin{smallmatrix} 1 \\ 10 \end{smallmatrix} \right $	8	9½
¼d.	$\left  \begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right $	4	4½
			<hr/>
			£733 0 1¼

(7) 243 at £2 4s. 7¼d.

			2
			<hr/>
			486
4s.	$\left  \begin{smallmatrix} 1 \\ 5 \end{smallmatrix} \right $	48	12
6d.	$\left  \begin{smallmatrix} 1 \\ 4 \end{smallmatrix} \right $	6	1 6
1d.	$\left  \begin{smallmatrix} 1 \\ 6 \end{smallmatrix} \right $	1	0 3
¼d.	$\left  \begin{smallmatrix} 1 \\ 4 \end{smallmatrix} \right $	5	0¾
			<hr/>
			£541 18 9¾

(8) 317 at £4 3s. 9½d.

			4
			<hr/>
			1268
2s. 6d.	$\left  \begin{smallmatrix} 1 \\ 5 \end{smallmatrix} \right $	39	12 6
1s. 3d.	$\left  \begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right $	19	16 3
317 halfpence		13	2½
			<hr/>
			£1328 1 11½

£2 multiplied by 37 gives £74; therefore a fifth part of £2, which is 8s., multiplied by 37, must give a fifth part of £74, that is, 14s. 6d.; so that 37 times 16s. is the double of 14s. 6d., or 14s. 6d. + 14s. 6d.; and 37 times the 6d. is 18s. 6d.: we thus get in successive rows 37 times the pounds, 37 times the shillings, and 37 times the pence; and the sum of these rows is therefore 37 times the proposed pounds, shillings, and pence.

In this Example, as well as in many Examples that may occur, it will be easier to compute for the *lowest* denomination (in the present case *pence*) by actual multiplication, instead of taking fractional parts; 37 sixpences is seen in a moment to be 18s. 6d., but this sum is not so readily seen to be the 16th part of £14 16s.

(9) 353 at £7 18s. 4½d.

			7
			2471
10s.	176	10	
5s.	88	5	
2s. 6d.	44	2	6
10d.	14	14	2
353 f.		7	4½
			£2794 19 0¼

(10) 417 at £3 9s. 2½d.

			3
			1251
of £1	5s.	104	5
	4s.	83	8
	2d.	3	9
	½d.	17	4½
			417s.
			£1442 19 10½

(11) 358 at £6 11s. 5½d.

			6
			2148
10s.	179		
1s. 3d.	22	7	6
2½d.	3	14	7
			£2353 2 1

(12) 519 at £7 19s. 10¾d.

			519 at £8.
			519 at 1¼d.
			8
			1¼d. 1¼ 129¾d.
			£4152
			Subt. 2 14s. 0¾d.
			12) 648¾
			54s. 0¾d.
			£4149 5 11¼

(13)

			£ s. d.
			0 16 4
			4
2 qrs.	3	5	4
4 lbs.		8	2
1 lb.			7
			1¾
			£3 14 2¾

(14)

			£ s. d.
			1 19 9
			2
1 qr.	3	7	6
7 lbs.		8	5¼
of 1 qr., 4 lbs.		2	1¼
2 lbs.		1	2
			7 ½
			£3 19 10

(15)

			£ s. d.
			2 3 7½ × 8
			10
			21 16 3
			3
of 1 yd., 1 ft.	65	8	9
6 in.	17	9	0
1 in.		14	6½
			7 3¼
			1 2½
			£84 0 9¼

(16) 17 yds. 2 ft. = 53 ft.

			s. d.
			53
			13
6 in.	159		
2 in.	53		
			689s.
3d.	13	3	
½d.	2	2½	
			8 10¼ for the 8 in.
			20) 713 3¼
			£35 13 3¾

$$(17) \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 0 \quad 3 \quad 8\frac{1}{4} \\ \hline 7 \end{array}$$

$$\begin{array}{l} 10 \text{ dwt.} \\ 1 \text{ dwt.} \\ 12 \text{ grs.} \\ 6 \text{ grs.} \end{array} \left| \begin{array}{r} \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{12} \\ \frac{1}{2} \end{array} \right| \begin{array}{r} 1 \quad 5 \quad 9\frac{3}{4} \\ 1 \quad 10 \\ 2\frac{1}{2} \text{ very nearly.} \\ 1 \\ \hline \frac{1}{2} \end{array}$$

$$\text{£}1 \quad 7 \quad 11\frac{1}{2}$$

$$(18) \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 1 \quad 12 \quad 6 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 11 \quad 7 \quad 6 \\ \hline 2 \\ 22 \quad 15 \quad 0 \\ 16 \quad 3 \\ \hline \text{£}23 \quad 11 \quad 3 \end{array}$$

$$3\frac{1}{2} \text{ days} \left| \frac{1}{2} \right|$$

(19) 3 months 3 weeks=15 weeks.

$$\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 1 \quad 3 \quad 10 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 5 \quad 19 \quad 2 \\ \hline 3 \end{array}$$

$$3 \text{ da.} \left| \frac{1}{2} \right| \begin{array}{r} 17 \quad 17 \quad 6 \\ 11 \quad 11 \end{array}$$

$$\text{£}18 \quad 9 \quad 5$$

(20)

127 at £3 6s. 8d.

$$\begin{array}{r} 3 \\ \hline 381 \\ 42 \quad 6 \quad 8 \\ 3 \quad 5 \quad 5 \\ \hline \text{£}426 \quad 12 \quad 1 \end{array}$$

2 roo.

1 roo.

20 per.

10 per.

5 per.

of 10p., 2p.

$$\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 3 \quad 6 \quad 8 \end{array}$$

$$\begin{array}{r} 1 \quad 13 \quad 4 \\ \hline 16 \quad 8 \end{array}$$

$$\begin{array}{r} 8 \quad 4 \\ \hline 4 \quad 2 \end{array}$$

$$\begin{array}{r} 2 \quad 1 \\ \hline 10 \end{array}$$

Value of 3 roo. 37 per.  $\text{£}3 \quad 5 \quad 5$

(21)

$$\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 5 \quad 5 \quad 0 \end{array}$$

$$\begin{array}{l} 10 \text{ cwt.} \\ 5 \text{ cwt.} \\ \text{of 10 cwt., 2 cwt.} \\ 2 \text{ grs.} \\ 1 \text{ qr.} \end{array} \left| \begin{array}{r} \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{2} \end{array} \right| \begin{array}{r} 2 \quad 12 \quad 6 \\ 1 \quad 6 \quad 3 \\ 10 \quad 6 \\ 2 \quad 7\frac{1}{2} \\ 1 \quad 3\frac{1}{2} \end{array}$$

$$\text{£}4 \quad 13 \quad 2\frac{1}{2}$$

(22) 86 lbs. 3 oz. = 1035 oz., at £4 16s. 4d.

$$\begin{array}{r} 4 \\ \hline \text{of £4, 16s.} \left| \frac{1}{10} \right| \begin{array}{r} 4140 \\ 828 \\ 17 \quad 5 = 1035s. \div 20 \times 3, \text{ for the 4d.} \\ \hline \text{£}4985 \quad 5 \text{ value of 86 lbs. 3 oz.} \end{array} \end{array}$$

$$\begin{array}{l} 10 \text{ dwt.} \\ 5 \text{ dwt.} \\ 12 \text{ grs.} \\ 6 \text{ grs.} \end{array} \left| \begin{array}{r} \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{2} \end{array} \right| \begin{array}{r} 4 \quad 16 \quad 4 \\ 2 \quad 8 \quad 2 \\ 1 \quad 4 \quad 1 \\ 2 \quad 4 \quad \frac{8}{10} \\ 1 \quad 2 \quad \frac{6}{20} \end{array}$$

$$\text{£}3 \quad 15 \quad 10\frac{1}{2} \text{ value of 15 dwt. 18 grs.}$$

$$\text{£}4989 \quad 0 \quad 10\frac{1}{2} \text{ whole value.}$$

(23)

		51 oz.	at		£	s.	d.
		3			3	8	4
		153	10 dwts.	$\frac{1}{2}$	1	14	2
		17	5 dwts.	$\frac{1}{2}$		17	1
6s. 8d.	$\frac{1}{2}$				£2	11	3
1s. 8d.	$\frac{1}{4}$	4	5				val. of 15 dwts.
		2	11	3			
		£176	16	3			val. of 15oz. 15 dwts.

(24) 43127 at £9 17s.

10

---

431270

2s. 6d. | 5390 17 6

6d. | 1078 3 6

---

*Subtract* 6469 1 0

---

£424800 19 0

(25) 
$$\begin{array}{r} 2623656 \\ 828138 \\ \hline 3451794 \text{ at } 7s. 10d. \\ 8 \\ \hline 27614352, \text{ duty at } 8s. \\ 575299, \text{ subd. (duty at } 2d.) \\ \hline 20)2703905,3 \\ \hline \pounds 1351952 \text{ } 13s. \end{array}$$

(26)		(27)	
This is the same as		£	s. d.
cwt.		Captain	10 0 0
13 at 6s. per stone.		Mate	5 0 0
14 no. of st. in 1 cwt.		Second mate	2 15 0
		Carpenter	4 0 0
52		Able seamen	36 0 0 = £2 5s. × 16
13			57 15 0
182	107 days. = 3m. 17 days.		3
5s. $\frac{1}{4}$ 45 10		10 days. $\frac{1}{3}$ 173 5 0	
1s. $\frac{1}{8}$ 9 2		5 days. $\frac{1}{2}$ 19 5 0	
£54 12s.	of 10 days, 2 days. $\frac{1}{8}$ 9 12 6		
		3 17 0	
		£205 19 6	

(28)

		2625	at 7d.	or	2625
6d.	1	1312	6		7
1		218	9	12)	18375
	2,0)	153,1	3	20)	153,1
		£76	11s. 3d.		3d.
		£76	11s. 3d.		£76 11s. 3d.





*Rule of Three; or, Simple Proportion,\* pages 106, 107.*

(1)

17 lbs. : 23 lbs. :: 11s. 7d. : 15s. 8½d., or, 11s. 7d. × 3

$$\begin{array}{r} 12 \\ 139 \\ 23 \\ \hline 417 \\ 278 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ 115\ 10 \\ 2 \\ \hline 231\ 8 \\ 34\ 9 \\ \hline \end{array}$$

17)3197(188½d. ∴ 12 = 15s. 8½d. 17)266s. 5d.(15s. 8½d.

$$\begin{array}{r} 17 \\ 149 \\ 136 \\ \hline 137 \\ 136 \\ \hline 1 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ 96 \\ 85 \\ \hline 11 \\ 12 \\ \hline 137 \\ 136 \\ \hline 1 \\ \hline \end{array}$$

(2) 9 lbs. : 112 lbs. :: £1 18s. 6d. : £23 19s. 1½d. + ½f.

$$\begin{array}{r} 8 \\ 15\ 8\ 0 \\ \hline 112 = 8 \times 7 \times 2 \\ 107\ 16\ 0 \\ \hline 2 \\ 9)215\ 12\ 0 \\ \hline \pounds 23\ 19\ 1\frac{1}{2}d. + \frac{1}{2}f. \end{array}$$

\* In working a Rule of Three question, the learner must never forget that the first and second terms of the stating, although they may be concrete numbers at the outset, are always used as purely abstract numbers, and that, as concrete numbers, they can never be employed. This has been explained at length in the Arithmetic. In working the first of the questions above, although it is quite correct to say that

17 lbs. : 23 lbs. :: 11s. 7d. : 15s. 8½d.,

yet it must be understood that the first two terms of the proportion are replaced by the abstract numbers 17 : 23 before either of them are actually used : it is plain that such a change in the character of these terms is quite allowable, because just as 17 lbs. is to 23 lbs., so is the

(3) 1 lb. :  $52\frac{3}{4}$  lbs.  $\times 6 = 316\frac{1}{2}$  lbs. ::  $5\frac{3}{4}$  d., or multiplying first and second terms by 2.

$$2 \text{ lbs.} : 633 \text{ lbs.} :: 5\frac{3}{4} \text{ d.} : £7 \text{ 11s. } 7\frac{3}{4} \text{ d.} + \frac{1}{2} \text{ f.}$$

$$\begin{array}{r} 23 \\ \hline 1899 \\ 1266 \\ \hline 2)14559 \\ 4)7279\frac{1}{2} \text{ f.} \\ 12)1819\frac{3}{4} \text{ d.} \\ 2,0)15,1 \text{ s. } 7 \text{ d.} \\ \hline £7 \text{ 11s. } 7\frac{3}{4} \text{ d.} + \frac{1}{2} \text{ f.} \end{array}$$

(4) As the number of persons must be greater if the number of days be fewer, the stating must be as follows:—

$$\begin{array}{rcll} & & \text{persons.} & \text{persons.} \\ (See \textit{Arith.}, p. 103) & 21 & : & 36 :: 28 : 48^* \\ & 7 & : & 12 \\ & 1 & : & 12 :: 4 \\ & & 4 & \\ & & \hline & & 48 & \text{persons, the No. required.} \end{array}$$

$$\begin{array}{rcll} & & \text{soldiers.} & \text{soldiers.} \\ (5) & 15 & : & 9 :: 1000 : 600 \\ & 5 & : & 3 \\ (\textit{Arith.}, p. 103) & 1 & : & 3 :: 200 \\ & & 3 & \\ & & \hline & & 600 & \text{soldiers 15 months.} \\ & & & \therefore 400 \text{ must be dismissed.} \end{array}$$

$$\begin{array}{rcll} & & \text{oz.} & \text{oz.} \\ (6) & 9 & : & 5 :: 12 : 6\frac{2}{3} \\ (\textit{Arith.}, p. 103) & 3 & : & 5 :: 4 \\ & & 5 & \\ & & \hline & & 3)20 & \\ & & & 6\frac{2}{3} \text{ oz.} \end{array}$$

number 17 to the number 23; in other words, if the above proportion be true, so also must the proportion

$$17 : 23 :: 11 \text{ s. } 7 \text{ d.} : 15 \text{ s. } 1\frac{1}{3} \text{ d.}$$

be true. And similar remarks apply to every exercise in proportion: concrete quantities can never be multiplied together.

\* In examples such as this, where the first two terms of the proportion are to stand in an order the reverse of that in which they

$$(7) \quad \begin{array}{ccccccc} \text{lbs.} & \text{cwt.} & \text{qrs.} & \text{lbs.} & \text{s.} & \text{d.} & \\ 14 & : & 1 & 3 & 17 & :: & 7 & 4 & : & £5 \text{ 11s. } 6\frac{3}{4}\text{d.} + \frac{1}{2}\text{f.} \end{array}$$

$$\begin{array}{r} 4 \\ 7 \\ 28 \\ \hline 213 \\ 7 \end{array}$$

$$4d. \left| \frac{1}{2} \right. \begin{array}{r} 1491 \\ 71 \end{array}$$

$$2)1562 \text{ shillings.}$$

$$14 = 2 \times 7 \quad 7) 781$$

$$20)11,1s. 6\frac{3}{4}\text{d.} + \frac{1}{2}\text{f.}$$

$$\underline{\underline{£5 \text{ 11s. } 6\frac{3}{4}\text{d.} + \frac{1}{2}\text{f.}}}$$

$$(8) \quad \begin{array}{ccc} \text{cwt.} & \text{qrs.} & \text{cwt.} \\ 1787 & 2 & : 19\frac{1}{2} :: £907 \text{ 10s.} : £9 \text{ 18s.} \\ \hline 2 & 2 & 20 \\ 3575 & : 39 & 18150 \end{array}$$

$$\begin{array}{r} 39 \\ \hline 163350 \\ 5445 \end{array}$$

If 3575 and 18150  
be each divided by  
5 × 5, the stating will  
be 143 : 39 :: 726  
or 11 : 3 :: 726  
3  
11)2178  
198s.

$$\begin{array}{r} 3575)707850(198s. \div 20 = £9 \text{ 18s.} \\ \underline{3575} \\ 35035 \\ \underline{32175} \\ 28600 \\ \underline{28600} \end{array}$$

occur in the question, the proportion is frequently called *inverse proportion*, and in some books on arithmetic, questions in simple proportion are accordingly divided into two separate classes, under the heads of "Rule of Three Direct" and "Rule of Three Inverse," in accordance with the plan so injudiciously followed in such books of making every operation merely mechanical, and independent of all thought and reflection on the part of the learner. Nobody who considers the question for a moment can require to be told whether it belongs to the Rule of Three Direct, or to the Rule of Three Inverse.

The learner may here be apprised—as the form of expression is often used—that in the above question the number of reapers is said to *vary inversely* as the number of days: in the preceding question the price *varies directly* as the weight. In question 5 the number of soldiers remaining in the garrison *varies inversely* as the number of months; and so on.

$$(9) \quad £2500 : £525 :: £112 2s. : £23 10s. 9\frac{3}{4}d.$$

$$\begin{array}{r} 100 : 21 \quad 20 \\ \hline 2242 \\ 21 \\ \hline 2242 \\ 4484 \\ \hline 470,82s. \\ 12 \\ \hline 9,84d. \\ 4 \\ \hline 3,36f. \end{array}$$

$\therefore$  he must pay 2,0)47,0s. 9 $\frac{3}{4}$ d., or £23 10s. 9 $\frac{3}{4}$ d.

$$(10) \quad \frac{5}{8} : \frac{3}{4} \times \frac{3}{7} :: £525 : £270$$

Or, multiplying first and second by 4,

$$\frac{5}{2} : \frac{3}{1} \times \frac{3}{7} :: £525$$

$$\text{that is } \frac{5}{2} : \frac{9}{7} :: £525$$

$$£525 \times \frac{9}{7} \div \frac{5}{2} = £ \frac{525 \times 9 \times 2}{7 \times 5} = £ \frac{105 \times 18}{7} =$$

$$£15 \times 18 = £270.$$

$$(11) \quad 1 \text{ oz.} : 106\frac{1}{2} \text{ s.} :: £3 6s. 8d. : £4240.$$

$$\begin{array}{r} 12 \quad 20 \\ 1272 \quad 66 \\ 800 \quad 12 \\ \hline 12)1017600 \quad 800 \\ 2,0)8480,0 \\ \hline £4240 \end{array}$$

(12)  $23 : 2978 :: 1000 : 129478$

23) 2978000 ( 129478 *amount of population.*

$$\begin{array}{r}
 23 \\
 \hline
 67 \\
 46 \\
 \hline
 218 \\
 207 \\
 \hline
 110 \\
 92 \\
 \hline
 180 \\
 161 \\
 \hline
 190 \\
 184 \\
 \hline
 6 \\
 \hline
 \end{array}$$

(13) As there must be a greater number of sovereigns than of guineas, the stating must be

*No. of guis. No. of sovs.*

$$\begin{array}{rcl}
 20 : 21 :: 44\frac{1}{2} : 46\frac{29}{40} \\
 \underline{2} \qquad \qquad \underline{2} \\
 40 : 21 :: 89 \\
 \qquad \qquad \underline{21} \\
 \qquad \qquad 89 \\
 \qquad \underline{178} \\
 40) 1869 \\
 \underline{46\frac{29}{40}} \text{ the numb. of sovs.}
 \end{array}$$

(14)  $144 : 175 :: 1 :$

*lb. avoird.*

$$\begin{array}{rcl}
 144) 175(1 & 2 & 11 \ 16 \\
 \underline{144} & & \\
 31 & & \text{or} \\
 \underline{12} & & 14 \text{ oz. } 11 \text{ dwt. } 16 \text{ grs.} \\
 372 & & \text{troy.} \\
 \underline{288} & & \\
 84 & & \\
 \underline{20} & & \\
 1680 & & \\
 \underline{1584} & & \\
 96 & & \\
 \underline{24} & & \\
 384 & & \\
 \underline{192} & & \\
 2304 & & \\
 \underline{144} & & \\
 864 & & \\
 \underline{864} & &
 \end{array}$$

troy.

$$(15) \quad 175 : 144 :: 12 \text{ lbs. } 1 \text{ oz. } 16 \text{ dwt. } 16 \text{ gr.} : 10 \text{ lbs. avoirdupois.}$$

$$\begin{array}{r}
 12 \\
 \hline
 145 \\
 \hline
 20 \\
 \hline
 2916 \\
 \hline
 24 \\
 \hline
 11670 \\
 \hline
 5833 \\
 \hline
 70000 \text{ grs. troy.} \\
 144 \\
 \hline
 5)10080000 \\
 175=5 \times 5 \times 7 \quad 5)2016000 \\
 \hline
 7)403200 \\
 \hline
 8)57600 \\
 24=8 \times 3 \quad 3)7200 \\
 \hline
 20)240,0 \\
 \hline
 12)120 \\
 \hline
 \text{weight of 1 imp. gal.} \quad 10 \text{ lbs. avoirdupois.} \\
 \text{of distilled water}
 \end{array}$$

In case the present question should seem perplexing to the learner, it may here be remarked that as 175 is to 144 so is any number of lbs. and fractions of a lb. troy, to the same expressed in lbs. and fractions of a pound avoirdupois; that is, the change from troy to avoirdupois weight is effected by multiplying the weight in lbs. troy by  $\frac{144}{175}$ ; and this is what is done above. For convenience, the given troy weight has been reduced to grains and then multiplied by the fraction  $\frac{144}{175}$ ; the result of this multiplication is 57600 grains troy, so that the troy weight in lbs. of *these* grains is expressed by the same number as the avoirdupois weight in lbs. of the original 70000 grains troy.

It follows, from the above, that 7000 grains troy = 1 lb. avoirdupois, and that 5760 grains troy = 1 lb. troy; therefore, to convert troy lbs. into avoirdupois, we may multiply by  $\frac{875}{1000}$ , which is a more convenient fraction than  $\frac{144}{175}$ , and derivable from it by multiplying numerator and denominator by 4.

If this form of the fraction had been employed above, then, after the reduction to grains, the work would have been as follows:—

$$\begin{array}{r}
 70000 \text{ grains troy} \\
 576 \\
 \hline
 7,00)403200 \text{ } 00 \\
 \hline
 8)57600 \\
 \hline
 3)7200 \\
 \hline
 20)240,0 \\
 \hline
 12)120 \\
 \hline
 10 \text{ lbs. avoirdupois}
 \end{array}$$

If lbs. avoirdupois are to be converted into lbs. troy, as in Example 14, then the multiplying fraction will of course be  $\frac{100}{81}$ .

(16) The space gone over by the short hand is always to the *gain* of the long hand as 1 to 11, and it is plain that when the hands are in opposition, between 7 and 8 o'clock, the long hand must be just as far past 1 as the short hand is past 7; so that while the short hand has moved this small space from 7, the long hand has *gained* upon it the whole space from 12 to 1, that is, one hour-space.

$$\therefore 11 : 1 :: 1 \text{ ho.} : 5 \text{ min. } 27\frac{3}{11} \text{ sec.}$$

$$\begin{array}{r} 11 \overline{) 60 \text{ min.}} \\ \underline{5 \text{ min.} \dots 5} \\ 60 \\ 11 \overline{) 300} \\ \underline{27\frac{3}{11} \text{ sec.}} \end{array}$$

$\therefore$  the time is 5 min.  $27\frac{3}{11}$  sec. past 7 o'clock.

(17) At 5 o'clock the long hand is at 12, and since it comes up to the short hand between 5 and 6, it must have *gained* upon the short hand 5 hour-spaces when it has arrived at it; therefore

$$11 : 1 :: 5 \text{ ho.} : 27 \text{ min. } 16\frac{4}{11} \text{ sec.}$$

$$\begin{array}{r} 60 \\ 11 \overline{) 300} \\ \underline{27 \text{ min. } 3} \\ 60 \\ 11 \overline{) 180} \\ \underline{16\frac{4}{11} \text{ sec.}} \end{array}$$

$\therefore$  the time is 27 min.  $16\frac{4}{11}$  sec. past 5 o'clock.

$$(18) \begin{array}{l} \text{Irish mi.} \quad \text{Eng. mi.} \\ 11 : 14 :: 57 : 72\frac{9}{11} \end{array}$$

$$\begin{array}{r} 14 \\ 228 \\ 57 \\ 11 \overline{) 798} \\ \underline{72\frac{9}{11} \text{ Eng. miles.}} \end{array}$$

$$(19) \quad 2500 \times 7\frac{3}{4} = 2500 \times \frac{31}{4} =$$

$$\begin{array}{r} 625 \times 31 \\ \underline{31} \\ 625 \\ 1875 \\ \underline{9375 \text{ sheets in 2500 copies.}} \end{array} \qquad \begin{array}{r} 24 \\ \underline{20} \\ 480 \text{ sheets in a ream.} \end{array}$$

<i>sheets.</i>	<i>sheets.</i>	
480 :	19375 :	15s. 6d. : £31 5s. 7½d.
96 :	3875	
	15	
	<hr/> 19375	
	3875	
6d. ½	1937 6	
	<hr/> 96)60062 6(625s. 7½d. ÷ 20 = £31 5s. 7½d.	
	576	<i>the cost for paper.</i>
	<hr/> 246	
	192	
	<hr/> 542	
	480	
	<hr/> 62	
	12	
	<hr/> 750	
	672	
	<hr/> 78	
	4	
	<hr/> 312	
	288	
	<hr/> 24	

This part of the operation  
may of course be performed  
by short division, since  $96 = 12 \times 8$ .

The stating above may be simplified by dividing the first and third terms by 3, and the work further reduced by proceeding thus : —

32 :	3875 ::	5s. 2d. : £31 5s. 7½d.
5s. ½	968 15	
2d. ½	32 5 10	
	<hr/> 4)1001 0 10	
	8)250 5 2½	
	<hr/> £31 5s. 7½d.	

(20) As the weight of the loaf increases as the price of wheat diminishes, the stating will be

$$64s. 3d. : 40s. 3d. :: 4 \text{ lbs.}$$



or,  $64\frac{1}{4} : 40\frac{1}{4} :: 4 \text{ lbs.} : 2 \text{ lbs. } 8\frac{3}{4} \text{ oz.}$

$$\begin{array}{r} \frac{4}{257} \quad \frac{4}{161} \times \frac{4}{257} \\ \hline \end{array}$$

257)644(2 lbs.  $8\frac{3}{4}$  oz., the weight of sixpenny loaf in 1830.

$$\begin{array}{r} 514 \\ 130 \\ 16 \\ \hline 780 \\ 13 \\ \hline 2080 \\ 2056 \\ \hline 24 \\ \hline \end{array}$$

(21) 5 sec. : 1 hour :: 36 yards :  $14\frac{8}{11}$  miles.

or, 1 sec. : 12 min.

$$\begin{array}{r} 60 \\ 720 \\ 36 \\ \hline 4320 \\ 216 \\ \hline \end{array}$$

No. of yds. in a mile, 176,0)25920( $14\frac{8}{11}$  miles, the hourly motion of the wind.

$$\begin{array}{r} 176 \\ 832 \\ 704 \\ \hline 128 \quad \frac{128}{176} = \frac{8}{11} \end{array}$$

(22) 7d. : £22 7s. 5d. ::  $\frac{\text{income.}}{\text{income.}}$  £1 : £767

$$\begin{array}{r} 20 \\ 447 \\ 12 \\ \hline 7)5369 \\ \hline \end{array}$$

£767, the income.

$$(23) \quad 66s. : 20s. :: 18 \text{ dwts.}$$

$$\text{or, } 11 : 20 :: 3 \text{ dwts.} : 5 \text{ dwts. } 10\frac{10}{11} \text{ grs.}$$

$$\begin{array}{r} 3 \\ 11 \overline{)60} \\ 5 \text{ dwt.} \dots 5 \\ \underline{24} \\ 11 \overline{)120} \\ 10\frac{10}{11} \text{ grs.} \end{array}$$

$\therefore$  there are 5 dwt.  $10\frac{10}{11}$  grs. of alloy in 20s.

$$\text{Again: } 66s. : 20s. :: 12 \text{ oz.} : 3 \text{ oz. } 12 \text{ dwt. } 17\frac{5}{11} \text{ grs.}$$

$$\text{or, } 11 : 20 :: 2 \text{ oz.}$$

$$\begin{array}{r} 2 \\ 11 \overline{)40} \\ 3 \text{ oz.} \dots 7 \\ \underline{20} \\ 11 \overline{)140} \\ 12 \text{ dwt.} \dots 8 \\ \underline{24} \\ 11 \overline{)192} \\ 17\frac{5}{11} \end{array}$$

$\therefore$  3 oz. 12 dwt.  $17\frac{5}{11}$  grs. is the weight of 20s.

$\frac{5}{11}$   $10\frac{10}{11}$  " weight of alloy.

3 oz. 7 dwt.  $6\frac{6}{11}$  grs. " weight of pure silver.

Or better thus:

12 oz. — 18 dwt. = 11 oz. 2 dwt., the weight of pure silver in 66s. : therefore

$$66s. : 20s. :: 11 \text{ oz. } 2 \text{ dwt.} : 3 \text{ oz. } 7 \text{ dwt. } 6\frac{6}{11} \text{ grs.}$$

$$\text{or, } 33 : 10 :: 222 \text{ dwt.}$$

$$\text{or, } 11 : 10 :: 74 \text{ dwt.}$$

$$\begin{array}{r} 10 \\ 11 \overline{)740} \\ 67 \text{ dwt.} \dots 3 \\ \underline{24} \end{array}$$

$\therefore$  3 oz. 7 dwt.  $6\frac{6}{11}$  grs. is the weight of pure silver.

$$\begin{array}{r} 11 \overline{)72} \\ 6\frac{6}{11} \text{ grs.} \end{array}$$

(24) 66s. weigh 1*lb. troy* (Example 23).

$$\therefore 66 : £275000 :: 1*lb. troy* :  $\frac{275000 \times 20}{66}$  *lbs. troy*.$$

And to convert these *lbs. troy* into *lbs. avoirdupois*, we must multiply by  $1\frac{1}{3}$  (see the work of Example 15) : so that the weight in *lbs. avoirdupois* is

$$\frac{11000 \quad 24}{275000 \times 20 \times 144} \text{ *lbs.* } = \frac{5280000}{11 \times 7} \text{ *lbs.*}$$

$$\frac{66 \times 175}{11 \quad 7}$$

$$\begin{array}{r} 11)5280000 \\ \underline{7)480000} \\ 4)68571\frac{1}{3} \text{ *lbs.* } \\ 4)17142 \quad 3 \end{array}$$

$$112 = 4 \times 4 \times 7$$

$$7)4285 \quad 11$$

$$20)61,2 \text{ cwt. } 27\frac{1}{3} \text{ *lbs.*}$$

$$\text{The weight } \underline{30} \text{ tons } 12 \text{ cwt. } 27\frac{1}{3} \text{ *lbs.*}$$

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### *Double Rule of Three, or Compound Proportion,* pages 111, 112.

(1) As the answer is to give the number of horses, the number of *horses* in the question is to be put for the common third term. This done, we may first consider the bushels and for the present disregard the days, or we may first disregard the bushels and take account only of the days : suppose we adopt the former plan, then as the greater number of bushels will feed the greater number of horses, the first two terms, to precede the third term just mentioned, will stand in this order—namely, 56 : 120.

Having thus arranged the bushels let us now look to the days, and as the fewer the number of days the greater the number of horses that can be kept on the same quantity of oats, the arrangement

of the days, in reference to the 14 horses in the third place, must be 24 : 16. Consequently the complete stating is

$$\begin{array}{l} 56 : 120 \\ 24 : 16 \end{array} \left. \vphantom{\begin{array}{l} 56 : 120 \\ 24 : 16 \end{array}} \right\} \begin{array}{l} \text{horses, horses.} \\ :: 14 : 20 \end{array}$$

and the wanting fourth term of this compound proportion is found by multiplying the third term 14, by  $120 \times 16$ , the product of the consequents, and dividing the result by  $56 \times 24$ , the product of the antecedents: that is to say, the number of horses required is

$$\frac{\begin{array}{ccc} 5 & 4 & 1 \\ 120 \times 16 \times 14 \end{array}}{\begin{array}{ccc} 56 & \times & 24 \\ 4 & & 1 \\ 1 \end{array}} = \frac{20}{1} = 20.$$

(2) As the answer is to be the number of *days*, the 9 days must be the third term of the compound proportion; and as a greater number of days of 10 hours, than days of 12 hours, will be necessary to travel any distance, the hours must be arranged thus: 10 : 12. Again, as 400 miles require more days than 250, the miles must be arranged thus, 250 : 400. Consequently the stating is

$$\begin{array}{l} 10 : 12 \\ 250 : 400 \end{array} \left. \vphantom{\begin{array}{l} 10 : 12 \\ 250 : 400 \end{array}} \right\} :: 9 \text{ days} : \frac{400 \times 12 \times 9}{250 \times 10} \text{ days} =$$

$\frac{8 \times 6 \times 9}{5 \times 5} \text{ days} = \frac{432}{25} \text{ days} = 17\frac{2}{25} \text{ days}$ , the number of days of 10 hours long.

(3) The answer here is to be *money*: therefore money, the 12*s.*, is to be the third term. As the money increases with the weight, the weights must be arranged thus: 2 *cut.* 3 *grs.* : 8 *cut.* 1 *gr.*; and as it diminishes with the distance, the distances must be arranged thus: 192 : 128. Consequently the compound proportion is as follows:

$$\begin{array}{l} 2 \text{ cut. } 3 \text{ grs.} : 8 \text{ cut. } 1 \text{ gr.} \\ 192 : 128 \end{array} \left. \vphantom{\begin{array}{l} 2 \text{ cut. } 3 \text{ grs.} : 8 \text{ cut. } 1 \text{ gr.} \\ 192 : 128 \end{array}} \right\} :: 12 \text{ s., or, } \begin{array}{l} 11 \text{ grs.*} : 33 \text{ grs.} \\ 192 : 128 \end{array} \left. \vphantom{\begin{array}{l} 11 \text{ grs.*} : 33 \text{ grs.} \\ 192 : 128 \end{array}} \right\} :: 12 \text{ s.} : 24 \text{ s.}$$

and we may now proceed as in the former Examples; but for the sake of variety, it may be as well to show here how the desired

\* The learner who remembers the observations on the nature of the operations in simple proportion at page 101 of the Arithmetic, need not be told here that the denomination *quarters* may be suppressed, and the numbers 11 and 33 regarded as purely *abstract numbers*: the denomination is inserted here that he may see, at a glance, how these

result may be reached by successive simplifications of the terms of the proportion. Dividing each of the first pair by 11, and each of the second pair by 64, we have

$$\left. \begin{array}{l} 1 : 3 \\ 3 : 2 \end{array} \right\} :: 12s., \text{ or, } \left. \begin{array}{l} 1 : 3 \\ 1 : 2 \end{array} \right\} :: 4s. : 4 \times 6s. = 24s. = £1 \ 4s.:$$

it being observed that we may always divide either of the antecedents and the third term by any factor common to both : in the present case 3 is a factor common to the antecedent 3 and the third term 12.

(4) . As the answer is to be *reams of paper*, the 66 reams must be the third term, and as more reams will be required for 5000 than for 3000 copies, the arrangement, as respects number of books, will be 3000 : 5000, or simply 3 : 5. Also, since a book of  $12\frac{1}{2}$  sheets will take more paper than one of only 11 sheets, the arrangement, as to these will be 11 :  $12\frac{1}{2}$ . Consequently, the compound proportion is

$$\left. \begin{array}{l} 3 : 5 \\ 11 : 12\frac{1}{2} \end{array} \right\} :: 66 \text{ reams} : \frac{66 \times 5 \times 12\frac{1}{2}}{3 \times 11} \text{ reams} =$$

$$\frac{2 \times 5 \times 12\frac{1}{2}}{1} \text{ reams} = 125 \text{ reams}$$

(5) The answer here is to be *days*; therefore the 36 days is to be the third term ; but as the required days are to be only 8 hours long, more of them will be occupied with the work than if they were 12 hours long : consequently, the arrangement as to the hours must be 8 : 12 or 2 : 3, and in reference to the workmen, the arrangement must be 30 : 24 or 5 : 4, because 30 will require fewer days than 24. Hence the compound proportion is

$$\left. \begin{array}{l} 2 : 3 \\ 5 : 4 \end{array} \right\} :: 36 \text{ days} : \frac{36 \times 3 \times 4}{2 \times 5} \text{ days} = \frac{18 \times 3 \times 4}{5} \text{ days} =$$

$$\frac{216}{5} \text{ days} = 43\frac{1}{5} \text{ days}.$$

(6) Here the answer is to be *days*, so that 168 days will form the third term; and since the days will be fewer for a larger number of soldiers, the arrangement for *them* will be 11268 : 939. As to the quarters of wheat, the greater the number of them the greater

numbers are obtained from the quantities above. There would be no *error* in putting the denominations in all cases; yet it is superfluous to introduce them, as, in the *work*, they are discarded : but it is proper to preserve the denomination of the third term, this denomination being always that of the answer, or fourth term of the proportion.

the number of days they will last; *they* therefore must be arranged thus, 351 : 1404; so that the compound proportion will be

$$\begin{aligned} 11268 : 939 \} & :: 168 \text{ days} : \frac{168 \times 939 \times 1404}{11268 \times 351} \text{ days} = \\ 351 : 1404 \} & \\ \frac{42 \times 313 \times 1404}{2817 \times 117} \text{ days} & = \frac{14 \times 313 \times 1404}{2817 \times 39} \text{ days} \\ & = \frac{14 \times 313 \times 468}{2817 \times 13} \text{ days} = 56 \text{ days.} \end{aligned}$$

$$\begin{array}{r} 468 \\ 313 \\ \hline 1404 \\ 468 \\ \hline 1404 \\ 2817 \quad 146484 \\ 13 \quad 14 \\ \hline 8451 \quad 585936 \\ 2817 \quad 146484 \\ \hline 36621 \quad 2050776 (56 \text{ days.} \\ 183105 \\ \hline 219726 \\ \hline 219726 \end{array}$$

•(7) The answer here is to be *weight*, so that the third term must be 32 oz. 8 dwt., or 648 dwt.; and as the weight increases as the price of wheat diminishes, the prices must be arranged thus, 54 : 60 or 9 : 10; also, since the weight of a loaf increases with the price of it, that of wheat being the same, the sixpence and eightpence must be arranged thus; 6 : 8 or 3 : 4. Consequently the compound proportion is

$$\begin{aligned} 9 : 10 \} & :: 648 \text{ dwt.} : \frac{648 \times 10 \times 4}{9 \times 3} \text{ dwt.} = \frac{72 \times 10 \times 4}{3} \text{ dwt.} \\ 3 : 4 \} & \\ & = 24 \times 10 \times 4 \text{ dwt.} = 960 \text{ dwt.} \end{aligned}$$

$$20 \overline{)960}$$

$$48 \text{ oz.} = 48 \div 16 \text{ lbs.} = 3 \text{ lbs.}, \text{ the weight required.}$$

(8) The answer here is to be *money*, so that £64 must be the third term; and as 13 persons spend more money than 12, all living at the same rate, the first antecedent and consequent are 13 : 12. Again, since the same number of persons would spend more in 9 months than in 8, the next antecedent and consequent are 8 : 9; and, finally, since the expenditure is the greater as the meat is dearer, the last antecedent and consequent are 6 : 6½. Hence the compound proportion is

$$\begin{aligned} 13 : 12 \} & \\ 8 : 9 \} & :: £64 : £ \frac{64 \times 12 \times 9 \times 6\frac{1}{2}}{13 \times 8 \times 6} = £ \frac{3 \times 2 \times 9 \times 6\frac{1}{2}}{13} \\ 6 : 6\frac{1}{2} \} & \\ & = £ \frac{144 \times 6\frac{1}{2}}{13} = £ \frac{936}{13} = £72, \text{ the expenditure required.} \end{aligned}$$

(9) As the answer is to be money, the third term must be money, namely, the £50 8s. 9d.; and since the larger farm pays the greater rent, the first pair of terms is 13 ac. 1 roo. 11½ per. : 8 ac. 3 roo. 22 per.; but as the value of the land increases with its quality, the next pair of terms must be 6 : 7. Therefore the compound proportion is

$$2131\frac{1}{2} : 1422 \left\{ \begin{array}{l} 6 : 7 \end{array} \right\} :: £50 \ 8s. \ 9d. : £39 \ 5s. \ 1\frac{1}{2}d.$$

$$\begin{array}{rcl} 13 \text{ a. } 1 \text{ roo. } 11\frac{1}{2} \text{ per.} & \frac{1422 \times 7}{2131\frac{1}{2} \times 6} & £50 \ 8s. \ 9d. \quad 8 \text{ a. } 3 \text{ roo. } 22 \text{ per.} \\ \underline{4} & & \underline{4} \\ 53 & & 35 \\ \underline{40} & \text{But the frac-} & \underline{40} \\ 2131\frac{1}{2} \text{ per.} & \text{tion is} & 1422 \text{ per.} \\ & = \frac{237 \times 7}{2131\frac{1}{2}} = \frac{237 \times 14}{4263} & \\ & = \frac{79 \times 14}{1421} = \frac{79 \times 2}{203} = \frac{158}{203} & \end{array}$$

therefore £50 8s. 9d. is to be multiplied by the fraction  $\frac{158}{203}$ ; and this may be done as follows:—

£	s.	d.	
50	8	9	× 8 = 403 10 0
		10	
504	7	6	× 5 = 2521 17 6
		10	
5043	15	0	× 1 = 5043 15 0
			203) 7969 2 6 ( £39 5s. 1½d., the rent required.
			609
			1879
			1827
			52
			20
			1042
			1015
			27
			12
			330
			203
			127
			4
			508
			406

(10) As the answer is to be *money*, the £7 10s. must be made the third term, and as 15 men can earn more than 12, time and wages being supposed the same, the arrangement for these numbers must be 15 : 12 or 5 : 4. Again, since  $18\frac{1}{2}$  days' work is worth more than 6 days' work, other things being the same, the days must be arranged thus, 6 :  $18\frac{1}{2}$ . Lastly, as a day of 9 hours is worth less than a day of 10 hours, the arrangement for the hours must be 10 : 9. Hence the compound proportion is

$$\left. \begin{array}{l} 5 : 4 \\ 6 : 18\frac{1}{2} \\ 10 : 9 \end{array} \right\} :: £7\frac{1}{2} : \frac{4 \times 18\frac{1}{2} \times 9}{5 \times 6 \times 10} £7\frac{1}{2}.$$

Dividing the terms of the fraction by  $4 \times 3$ , it becomes

$$\frac{18\frac{1}{2} \times 3}{5 \times 5} = \frac{55}{5 \times 5} = \frac{11}{5} : \text{and } \frac{11}{5} \times \frac{15}{2} £ = £ \frac{11 \times 3}{2} = £16\frac{1}{2} \\ = £16 \text{ 10s., the wages required.}$$

## DECIMALS.

*Reduction of Common Fractions to Decimals, page 117.*

$$\begin{array}{r} (1) \quad 4 \overline{)7} \\ \quad 4 \overline{)1.75} \\ \hline \therefore \frac{7}{16} = .4375 \end{array}$$

$$\begin{array}{r} (2) \quad 8 \overline{)5} \\ \quad 8 \overline{).625} \\ \hline \therefore \frac{5}{8} = .625 \end{array}$$

$$\begin{array}{r} (3) \quad 5 \overline{)67} \\ \quad 5 \overline{)13.4} \\ \quad 5 \overline{)2.68} \\ \hline \therefore \frac{67}{125} = .536 \end{array}$$

$$\begin{array}{r} (4) \quad 5 \overline{)93} \\ \quad 5 \overline{)18.6} \\ \quad 10 \overline{)3.72} \\ \hline \therefore \frac{93}{250} = .372 \end{array}$$

$$\begin{array}{r} (5) \quad 5 \overline{)4} \\ \quad 5 \overline{).8} \\ \quad 5 \overline{).16} \\ \quad 5 \overline{).032} \\ \hline \therefore \frac{4}{625} = .0064 \end{array}$$

$$\begin{array}{r} (6) \quad \frac{14}{2125} = \frac{56}{12500} = .56 \div 125. \\ \quad 5 \overline{)56} \\ \quad 5 \overline{)112} \\ \quad 5 \overline{)224} \\ \hline \therefore \frac{14}{3125} = .00448 \end{array}$$

$$(7) \quad \frac{10\frac{1}{2}}{20} = \frac{10.75}{20} = \frac{1.075}{2} = .5375.$$



$$(8) \quad \frac{4\frac{1}{8}}{758} = \frac{4.2}{758} = \frac{2.1}{379} = .005541.$$

$$379)2.1 \quad (.005541 \text{ (See p. 116 Arith.)})$$

$$\begin{array}{r} 1895 \\ \underline{205} \\ 1895 \\ \underline{155} \\ 1516 \\ \underline{34} \end{array}$$

The number of decimals used in the dividend of Ex. 8 is *six*, for the dividend has been taken 2.100000; and as the number of decimals in divisor and quotient together must equal the number used in the dividend, six places must be pointed off in the quotient.

$$(9) \quad \frac{3}{5} \times \frac{8}{7} = \frac{24}{35} = .6857143$$

$$\begin{array}{r} 5)24 \\ 7)4.8 \\ \hline \therefore \frac{3}{5} \text{ of } \frac{8}{7} = \underline{.6857143} \end{array}$$

$$(10) \quad \frac{2}{3} \times \frac{2\frac{1}{2}}{3\frac{1}{2}} = \frac{2}{3} \times \frac{10}{13} = \frac{20}{39} = .51282$$

$$\begin{array}{r} 39)20 \quad (.51282 \\ \underline{195} \\ 5 \\ \underline{39} \\ 11 \\ \underline{78} \\ 32 \\ \underline{312} \\ 8 \end{array}$$

$$(11) \quad \frac{4}{5} \times \frac{11}{12} = \frac{44}{60} = \frac{4.4}{6} = .733333, \text{ \&c.}$$

$$(12) \quad \frac{3}{8} \times \frac{2}{5} \times 6 = \frac{3}{2} \times \frac{1}{5} \times 3 = \frac{9}{10} = .9$$

*Addition and Subtraction of Decimals, page 118.*

$$\begin{array}{r} (1) \quad 27.62 \\ \quad .358 \\ \quad 17.3 \\ \quad 61 \\ \quad .007 \\ \hline 173.1 \\ \hline 279.385 \end{array}$$

$$\begin{array}{r} (2) \quad 5862.93 \\ \quad 38.041 \\ \quad 1.01 \\ \quad 176.4 \\ \quad .0004 \\ \hline 265.04 \\ \hline 6343.4214 \end{array}$$

$$\begin{array}{r}
 (3) \quad 385\cdot02 \quad 7\cdot03 \\
 \quad 18\cdot176 \quad 11\cdot11 \\
 \quad \quad 5\cdot328 \quad 21\cdot625 \\
 \quad \quad \quad \cdot061 \quad \underline{39\cdot765} \\
 408\cdot585 \\
 \underline{39\cdot765} \text{ to be subtracted.} \\
 \underline{368\cdot82} \text{ result.}
 \end{array}$$

$$\begin{array}{r}
 (4) \quad 1\cdot0628 \quad 26\cdot04 \\
 \quad 123\cdot51 \quad 18\cdot261 \\
 \quad \quad 13 \quad -\cdot082 \\
 \quad \quad \underline{12\cdot403} \quad \underline{44\cdot383} \\
 149\cdot9758 \\
 \underline{44\cdot383} \text{ to be subtracted.} \\
 \underline{105\cdot5928} \text{ result.}
 \end{array}$$

$$\begin{array}{r}
 (5) \quad \cdot623 \quad \cdot31 \\
 \quad \cdot0042 \quad \cdot002 \\
 \quad \cdot79 \quad \cdot0003 \\
 \quad \cdot11 \quad \underline{\cdot3123} \\
 \quad \cdot08 \\
 \underline{1\cdot6072} \\
 \underline{\cdot3123} \text{ to be subtracted.} \\
 \underline{1\cdot2949} \text{ result.}
 \end{array}$$

$$\begin{array}{r}
 (6) \quad 246 \quad 5\cdot613 \\
 \quad 187 \quad 19\cdot148 \\
 \quad \quad \cdot0018 \quad 7\cdot03 \\
 \quad \underline{433\cdot0018} \quad 104\cdot6 \\
 \quad 136\cdot391 \text{ subtract.} \quad \underline{136\cdot391} \\
 \underline{226\cdot6108} \text{ result.}
 \end{array}$$

~~~~~

*Multiplication of Decimals, page 119.*

$$\begin{array}{r}
 (1) \quad 32\cdot605 \\
 \quad \quad 6\cdot417 \\
 \quad \quad \underline{195630} \\
 \quad 130420 \\
 \underline{195630} \\
 \text{Product } \underline{209\cdot226285}
 \end{array}$$

$$\begin{array}{r}
 (2) \quad 183\cdot52 \\
 \quad \quad \cdot734 \\
 \quad \quad \underline{73408} \\
 \quad \quad 55056 \\
 \quad \quad \underline{128464} \\
 \text{Product } \underline{134\cdot70368}
 \end{array}$$

$$\begin{array}{r}
 (3) \quad 43\cdot92 \\
 \quad \quad 2600 \\
 \quad \quad \underline{2635200} \\
 \quad \quad 8784 \\
 \text{Product } \underline{114192}
 \end{array}$$

$$\begin{array}{r}
 (4) \quad \cdot038 \\
 \quad \cdot072 \\
 \quad \quad 76 \\
 \quad \quad \underline{266} \\
 \text{Prod. } \underline{\cdot002736}
 \end{array}$$

$$\begin{array}{r}
 (5) \quad \cdot0037 \\
 \quad \cdot00021 \\
 \quad \quad 37 \\
 \quad \quad \underline{74} \\
 \text{Prod. } \underline{\cdot000000777}
 \end{array}$$

$$\begin{array}{r}
 (6) \quad 2\cdot46 \\
 \quad \cdot321 \\
 \quad \quad 246 \\
 \quad \quad 492 \\
 \quad \quad \underline{738} \\
 \quad \quad \cdot78966 \\
 \quad \quad \underline{\cdot07} \\
 \text{Prod. } \underline{\cdot0552762}
 \end{array}$$

|     |                                                                                                                                                                         |     |                                                                                                                                                                       |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (7) | $  \begin{array}{r}  1.73 \\  .032 \\  \hline  346 \\  519 \\  \hline  .05536 \\  .0105 \\  \hline  27680 \\  5536 \\  \hline  \text{Prod. } .000581280  \end{array}  $ | (8) | $  \begin{array}{r}  24000 \\  .0016 \\  \hline  144000 \\  24 \\  \hline  38.4000 \\  .35 \\  \hline  1920 \\  1152 \\  \hline  \text{Prod. } 13.440  \end{array}  $ |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|     |                                                                                                                                                                                                            |
|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (9) | $  \begin{array}{r}  2.016 \\  3.004 \\  \hline  8064 \\  6048 \\  \hline  6.056064 \\  .0756 \\  \hline  36336384 \\  30280320 \\  \hline  42392448 \\  \hline  \text{Prod. } .4578384384  \end{array}  $ |
|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

The second multiplication in this 9th Example may be performed with fewer figures by adopting the principle explained in the footnote at p. 119 of the Arithmetic, thus :—

|                                                                                                                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $  \begin{array}{r}  6.056064 \\  .0756 \\  \hline  42392448 \text{ prod. by the 7} \\  339139584 \text{ last prod. by 8, for the 56} \\  \hline  \text{Prod. } 4578384384  \end{array}  $ |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|      |                                                                                                                                               |      |                                                                                                                                                                                      |      |                                                                                                                                       |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|---------------------------------------------------------------------------------------------------------------------------------------|
| (10) | $  \begin{array}{r}  273.4 \\  .036 \\  \hline  16404 \\  8202 \\  \hline  9.8424 \\  .004 \\  \hline  \text{Prod. } .0393696  \end{array}  $ | (11) | $  \begin{array}{r}  21000 \\  1.02 \\  \hline  42000 \\  21 \\  \hline  21420.00 \\  .0268 \\  \hline  171360 \\  12852 \\  4284 \\  \hline  \text{Prod. } 574.0560  \end{array}  $ | (12) | $  \begin{array}{r}  1.4 \\  .04 \\  \hline  .056 \\  .4 \\  \hline  .0224 \\  .004 \\  \hline  \text{Prod. } .000896  \end{array}  $ |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|---------------------------------------------------------------------------------------------------------------------------------------|

# SS KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

$$\begin{array}{r}
 (13) \quad 71 \cdot 380164 \\
 \quad \quad 2 \cdot 7354 \\
 \hline
 \quad \quad 285520656 \\
 \quad \quad 356900820 \\
 \quad \quad 214140492 \\
 \quad \quad 499661148 \\
 \quad \quad 142760328 \\
 \hline
 \text{Prod. } 195 \cdot 2533006056
 \end{array}$$

For another mode of working Example 13 see page 120 of the Arithmetic.

$$\begin{array}{r}
 (14) \quad 138 \cdot 6147 \\
 \quad \quad 5 \cdot 2575 \\
 \hline
 \quad \quad 6930735 \\
 \quad \quad 9703029 \\
 \quad \quad 6930735 \\
 \quad \quad 2772294 \\
 \quad \quad 6930735 \\
 \hline
 \quad \quad 728 \cdot 76678525 \\
 \quad \quad \quad .03 \\
 \hline
 \text{Prod. } 21 \cdot 8630035575
 \end{array}$$

Or thus (foot-note p. 119, Arith.)

$$\begin{array}{r}
 138 \cdot 6147 \\
 \quad 5 \cdot 2575 \\
 \hline
 6930735 \quad \text{prod. by 5} \\
 34653675 \quad \text{last prod. by 5 for 25} \\
 103961025 \quad \text{last prod. by 3 for 75} \\
 \hline
 728 \cdot 76678525 \\
 \quad .03 \\
 \hline
 \text{Prod. } 21 \cdot 8630035575
 \end{array}$$

## Contracted Multiplication of Decimals, page 125.

(1) As only four decimals are to be retained in the product, the units figure, 2, of the multiplier must be placed under the fourth decimal, 3, of the multiplicand, and the whole multiplier reversed.

$$\begin{array}{r}
 480 \cdot 14936 \\
 \quad 614272 \\
 \hline
 9602987 \\
 3361046^* \\
 96030 \\
 19206 \\
 480 \\
 288 \\
 \hline
 \text{Prod. } 1308 \cdot 0037
 \end{array}$$

\* If only the single back-figure 8 be looked to, to supply the carrying, the first figure here will be 5 instead of 6; but if the preceding back-figure 6 be also taken into account, the carrying will be from 36 and not from 3 merely, and the first figure to be put down will then be 6 as above: it is in general the surest way to examine in this manner the influence of more than a single back-figure when the utmost degree of accuracy in the final decimal of the product is of consequence.

(2) As the final decimal of each factor in this Example is supposed to be erroneous, and as the factor which has the greater number of figures has *eight*, eight decimal places must be discarded from the complete, or uncontracted product; and since this complete product would have 11 decimals, we limit the result to *three* decimals, as all that can safely be depended upon, and proceed in either of the following ways:—

|               |          |
|---------------|----------|
| 15·917127     | 30·31667 |
| 7661303       | 72171951 |
| 477514        | 303167   |
| 4775          | 151583   |
| 159           | 27285    |
| 96            | 303      |
| 9             | 212      |
| 1             | 3        |
| Prod. 482·554 | 482·554  |

(3) In this Example, if we proceed according to the first mode of working given below, estimating the carryings from only a single back figure in each case, and then according to the second mode, estimating the carryings from examining the two back figures, in each case there will be a little difference in the resulting fourth decimal. As remarked in the book, when the final decimal retained is of importance an *additional* decimal place should always be at first computed and then rejected, as already explained (pp. 122, 126):

|               |          |
|---------------|----------|
| 30·31667      | 30·31667 |
| 36585971      | 36585971 |
| 303167        | 303167   |
| 212216        | 212217   |
| 27284         | 27285    |
| 1516          | 1516     |
| 242           | 242      |
| 15            | 15       |
| 2             | 2        |
| Prod. 54·4442 | 54·4444  |

(4)

|                  |
|------------------|
| ·62311052        |
| 071              |
| 62311052         |
| 43617736         |
| Prod. 105·928788 |

(5) Computing to one place of decimals, the operation is

|             |
|-------------|
| 1·628894    |
| 78412       |
| 3258        |
| 163         |
| 65          |
| 13          |
| 1           |
| Prod. 350·0 |

(6) Taking account of the two back figures, in each case, for the carryings, the work will be as follows :—

$$\begin{array}{r}
 81\cdot4632 \\
 156427 \\
 \hline
 570242 \\
 16293 \\
 3259 \\
 489 \\
 41 \\
 1 \\
 \hline
 \text{Prod. } 590\cdot325
 \end{array}$$

But if we compute to an additional decimal, regarding only a single back figure, it will stand thus :—

$$\begin{array}{r}
 81\cdot4632 \\
 156427 \\
 \hline
 5702424 \\
 162926 \\
 32585 \\
 4888 \\
 407 \\
 8 \\
 \hline
 \text{Prod. } 590\cdot324
 \end{array}$$

(7) Computing to six decimals, taking account of the two back figures, in the carryings, the work is as follows :—

$$\begin{array}{r}
 3\cdot7719214 \\
 81617440 \\
 \hline
 1508769 \\
 150877 \\
 26403 \\
 377 \\
 226 \\
 4 \\
 3 \\
 \hline
 \text{Product } 1\cdot686659
 \end{array}$$

If we compute to an additional decimal, regarding only a single back figure in each multiplication, the work will be as below; the additional decimal being dismissed at the close.

$$\begin{array}{r}
 3\cdot7719214 \\
 81617440 \\
 \hline
 15087686 \\
 1508768 \\
 264034 \\
 3772 \\
 2263 \\
 38 \\
 30 \\
 \hline
 \text{Product } 1\cdot686659
 \end{array}$$

(8) As each factor contains six decimals, six is the greatest number that can be depended upon in the product. The computation for six decimals is as follows :—

$$\begin{array}{r}
 \cdot053407 \\
 6217400 \\
 \hline
 2136 \\
 374 \\
 5 \\
 1 \\
 \hline
 \text{Prod. } \cdot002516
 \end{array}$$

If we include an additional decimal and compute to seven places, still, however, retaining only six in the product, the work will be

$$\begin{array}{r}
 \cdot0534070 \\
 6217400 \\
 \hline
 21363 \\
 3738 \\
 53 \\
 11 \\
 3 \\
 \hline
 \text{Prod. } \cdot002517
 \end{array}$$

$$(9) \begin{array}{r} 325\cdot701428 \\ 39381270 \\ \hline \end{array}$$

$$\begin{array}{r} 227991 \\ 6514 \\ 326 \\ 261 \\ 10 \\ 3 \\ \hline \end{array}$$

$$\text{Prod. } \underline{\underline{235\cdot105}}$$

Or more accurately, by computing an additional decimal, as follows:—

$$\begin{array}{r} 325\cdot701428 \\ 39381270 \\ \hline 2279910 \\ 65140 \\ 3257 \\ 2606 \\ 98 \\ 29 \\ 1 \\ \hline \end{array}$$

$$\text{Prod. } \underline{\underline{235\cdot104}}$$

(10) As there are five figures in each factor, no more than  $10 - 5 = 5$  decimals in the product can be depended upon: hence, computing to six decimals, and discarding the last of them in adding up the partial products, the work will stand as follows:—

$$\begin{array}{r} \cdot63942 \\ 71235 \\ \hline 319710 \\ 19183 \\ 1279 \\ 64 \\ 45 \\ \hline \end{array}$$

$$\text{Prod. } \underline{\underline{\cdot34028}}$$

*Division of Decimals, page 129.*

$$(1) \begin{array}{r} 5)267\cdot15975 \\ 5)5343195 \\ 5)1068639 \\ \hline 213\cdot728 \end{array}$$

$$(2) \begin{array}{r} \cdot0325)5\cdot474558 (168\cdot448 \\ \underline{325} \\ 2224 \\ \underline{1950} \\ 2745 \\ \underline{2600} \\ 1455 \\ \underline{1300} \\ 1558 \\ \underline{1300} \\ 258 \\ \underline{2600} \end{array}$$

92 KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

- |                                                                                                                                                                             |                                                                                                                                                                                                             |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>(3) 1·0125)325 (320·988</p> $  \begin{array}{r}  30375 \\  \hline  2125 \\  20250 \\  \hline  1000 \\  91125 \\  \hline  8875 \\  81000 \\  \hline  7750  \end{array}  $ | <p>(4) 7849)519·62 (.066</p> $  \begin{array}{r}  47094 \\  \hline  4868 \\  47094 \\  \hline  1586  \end{array}  $                                                                                         |
| <p>(5) 6·029)47·298 (7·845</p> $  \begin{array}{r}  42203 \\  \hline  5095 \\  48232 \\  \hline  2718 \\  24116 \\  \hline  3064  \end{array}  $                            | <p>(6) ·09317)3176·82 (34097·027</p> $  \begin{array}{r}  27951 \\  \hline  38172 \\  37268 \\  \hline  904 \\  83853 \\  \hline  6547 \\  65219 \\  \hline  251 \\  18634 \\  \hline  6466  \end{array}  $ |

*Contracted Division, page 132.*

- |                                                                                                                                                                    |                                                                                                                                                                                                                                        |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>(1) 4·817)31·782 (6·598</p> $  \begin{array}{r}  \dots 28902 \\  \hline  2880 \\  2409 \\  \hline  471 \\  433 \\  \hline  38 \\  38 \\  \hline  \end{array}  $ | <p>(2) ·573286)2490·3048 (4343·91</p> $  \begin{array}{r}  \dots 2293144 \\  \hline  197161 \\  171986 \\  \hline  25175 \\  22931 \\  \hline  2244 \\  1720 \\  \hline  524 \\  516 \\  \hline  8 \\  6 \\  \hline  2  \end{array}  $ |
| <p>(3) 500·78)2·149 (.00429</p> $  \begin{array}{r}  \dots 2003 \\  \hline  146 \\  100 \\  \hline  46 \\  45 \\  \hline  1  \end{array}  $                        |                                                                                                                                                                                                                                        |



(4)  $6\cdot029)47\cdot298(7\cdot845$

$$\begin{array}{r} \dots 42203 \\ \hline 5095 \\ 4823 \\ \hline 272 \\ 241 \\ \hline 31 \\ 30 \\ \hline 1 \\ \hline \end{array}$$

(5)  $325)4650\cdot75(14\cdot31$

$$\begin{array}{r} 325 \\ \hline 1400 \\ 1300 \\ \hline 1007 \\ 975 \\ \hline 325 \\ 325 \\ \hline \end{array}$$

(6)  $7\cdot3524)8\cdot6134(1\cdot1715$

$$\begin{array}{r} \dots 73524 \\ \hline 12610 \\ 7352 \\ \hline 5258 \\ 5146 \\ \hline 112 \\ 74 \\ \hline 38 \\ 37 \\ \hline 1 \\ \hline \end{array}$$

(7)  $3\cdot142)16\cdot804,379(5\cdot348$

$$\begin{array}{r} \dots 15710 \\ \hline 1094 \\ 943 \\ \hline 151 \\ 126 \\ \hline 25 \\ 25 \\ \hline \end{array}$$

(8)  $\cdot41432)673\cdot14,89(1624\cdot7$

$$\begin{array}{r} \dots 41432 \\ \hline 25883 \\ 24859 \\ \hline 1024 \\ 829 \\ \hline 195 \\ 166 \\ \hline 29 \\ 29 \\ \hline \end{array}$$

(9)  $3\cdot1415927)2\cdot7182818(865256$

$$\begin{array}{r} \dots 25132742 \\ \hline 2050076 \\ 1884955 \\ \hline 165121 \\ 157080 \\ \hline 8041 \\ 6283 \\ \hline 1758 \\ 1571 \\ \hline 187 \\ 188 \\ \hline \end{array}$$

94 KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

|      |                        |      |                           |
|------|------------------------|------|---------------------------|
| (10) | 8.1,92)·00128(·000156* | (11) | ·00024)170,84592(71,00000 |
|      | 82                     |      | 168                       |
|      | 46                     |      | 3                         |
|      | 41                     |      | 2                         |
|      | 5                      |      | 1                         |
|      | 5                      |      |                           |
|      | —                      |      | —                         |

|      |                   |      |                   |
|------|-------------------|------|-------------------|
| (12) | 847,36)3412(·0403 | (13) | ·3829)7534,7(1968 |
|      | 3389              |      | 3829              |
|      | 23                |      | 3706              |
|      | 25                |      | 3446              |
|      |                   |      | 260               |
|      |                   |      | 230               |
|      |                   |      | 30                |
|      |                   |      | 31                |
|      |                   |      | —                 |

|      |            |      |                                     |
|------|------------|------|-------------------------------------|
| (14) | 10473654)1 | (15) | ·09547766 3½ = 3·25)5474558(1684479 |
|      | 94262886   |      | 325                                 |
|      | 5737114    |      | 2224                                |
|      | 5236827    |      | 1950                                |
|      | 500287     |      | 2745                                |
|      | 418946     |      | 2600                                |
|      | 81341      |      | 1455                                |
|      | 73316      |      | 1300                                |
|      | 8025       |      | 1558                                |
|      | 7332       |      | 1300                                |
|      | 693        |      | 258                                 |
|      | 628        |      | 228                                 |
|      | 65         |      | 30                                  |
|      | 63         |      | 29                                  |
|      | 2          |      | 1                                   |
|      | —          |      | —                                   |

\* If we had stopped the operation at the preceding figure, that figure would have been a 6; it is more accurate however to make it a 5, and to put 6 for the closing decimal of the quotient: this 6 is multiplied by the 8 of the divisor, and only what would be *carried* from the result put under the last remainder: and the like is always to be done whenever we extend the figures of the quotient to the utmost allowable limit, but it must be observed that the last figure of the quotient, thus obtained, cannot, in general, be depended upon as true to the nearest unit.

$$\begin{array}{r}
 (16) \quad \cdot 1045 \overline{) 125} \quad (1 \cdot 196 \\
 \quad \quad \quad \dots \quad 1045 \\
 \quad \quad \quad \hline
 \quad \quad \quad 205 \\
 \quad \quad \quad 105 \\
 \quad \quad \quad \hline
 \quad \quad \quad 100 \\
 \quad \quad \quad 94 \\
 \quad \quad \quad \hline
 \quad \quad \quad 6 \\
 \quad \quad \quad 6 \\
 \quad \quad \quad \hline
 \quad \quad \quad -
 \end{array}$$

$$\begin{array}{r}
 (17) \\
 4 \overline{) 3} \\
 4 \overline{) \cdot 75} \\
 \frac{3}{16} = \cdot 1875 \therefore 23 \frac{3}{16} = 23 \cdot 1875, \\
 \text{and the operation of dividing} \\
 \text{this by } 87 \cdot 64378 \text{ is as follows:} \\
 87 \cdot 64378 \overline{) 23 \cdot 1875} \quad ( \cdot 2645653 \\
 \quad \dots \quad 17528756
 \end{array}$$

$$\begin{array}{r}
 (18) \quad \cdot 7854 \overline{) 14 \cdot 3589} \quad (18 \cdot 28 \\
 \quad \quad \quad \dots \quad 7854 \\
 \quad \quad \quad \hline
 \quad \quad \quad 6505 \\
 \quad \quad \quad 6283 \\
 \quad \quad \quad \hline
 \quad \quad \quad 222 \\
 \quad \quad \quad 157 \\
 \quad \quad \quad \hline
 \quad \quad \quad 65 \\
 \quad \quad \quad 63 \\
 \quad \quad \quad \hline
 \quad \quad \quad 2 \\
 \quad \quad \quad \hline
 \quad \quad \quad -
 \end{array}$$

$$\begin{array}{r}
 5658744 \\
 5258627 \\
 \hline
 400117 \\
 350575 \\
 \hline
 49542 \\
 43822 \\
 \hline
 5720 \\
 5259 \\
 \hline
 461 \\
 438 \\
 \hline
 23 \\
 \hline
 \hline
 \end{array}$$

(19)

Or thus :

$$\begin{array}{r}
 31773 \cdot 244 \overline{) 2972160} \quad (93 \cdot 54286 \\
 \dots \quad \quad 2859592 \\
 \quad \quad \quad \hline
 \quad \quad \quad 112568 \\
 \quad \quad \quad 95320 \\
 \quad \quad \quad \hline
 \quad \quad \quad 17248 \\
 \quad \quad \quad 15887 \\
 \quad \quad \quad \hline
 \quad \quad \quad 1361 \\
 \quad \quad \quad 1271 \\
 \quad \quad \quad \hline
 \quad \quad \quad 90 \\
 \quad \quad \quad 63 \\
 \quad \quad \quad \hline
 \quad \quad \quad 27 \\
 \quad \quad \quad 25 \\
 \quad \quad \quad \hline
 \quad \quad \quad 2 \\
 \quad \quad \quad 2 \\
 \quad \quad \quad \hline
 \quad \quad \quad -
 \end{array}$$

$$\begin{array}{r}
 31773 \cdot 244 \overline{) 2972160 \cdot 00} \quad (93 \cdot 54286 \\
 \dots \quad \quad 285959196 \\
 \quad \quad \quad \hline
 \quad \quad \quad 11256804 \\
 \quad \quad \quad 9531973 \\
 \quad \quad \quad \hline
 \quad \quad \quad 1724831 \\
 \quad \quad \quad 1588662 \\
 \quad \quad \quad \hline
 \quad \quad \quad 136169 \\
 \quad \quad \quad 127093 \\
 \quad \quad \quad \hline
 \quad \quad \quad 9076 \\
 \quad \quad \quad 6355 \\
 \quad \quad \quad \hline
 \quad \quad \quad 2721 \\
 \quad \quad \quad 2542 \\
 \quad \quad \quad \hline
 \quad \quad \quad 179 \\
 \quad \quad \quad 191 \\
 \quad \quad \quad \hline
 \quad \quad \quad -
 \end{array}$$

93 KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

(20) 1059·108)103·936(·098135 (21) 3·142)16·804379(5·348306  
 .... 95320 ... 15710

8616  
8473  
 143  
106  
 37  
32  
 5  
5

10943  
9426  
 15177  
12568  
 26099  
25136  
 963  
943  
 20  
19  
 1

(22) 1059·108)103·936 (·0981354

..... 9531972  
861628  
847286  
 14342  
10591  
 3751  
3177  
 574  
530  
 44  
42  
 2

(23) 500·78)2·149 (·0042913 &c. 8·4736)·3412 (·040266 &c.

200312

14588  
100156  
 45724  
450702  
 6538  
50078  
 15302  
150234  
2786

338944

2256  
169472  
 56128  
508416  
 52864  
508416  
20224

|      |                  |                  |                           |                   |
|------|------------------|------------------|---------------------------|-------------------|
|      | <i>cubic in.</i> | <i>cubic in.</i> | <i>old gals.</i>          | <i>imp. gals.</i> |
| (24) | 277·274          | : 231            | :: 63                     | : 52·486          |
|      |                  | 63               |                           |                   |
|      |                  | 693              |                           |                   |
|      |                  | 1386             |                           |                   |
|      | 277·274          | 14553            | (52·486 imperial gallons. |                   |
|      | ....             | 1386370          |                           |                   |
|      |                  | 68930            |                           |                   |
|      |                  | 55455            |                           |                   |
|      |                  | 13475            |                           |                   |
|      |                  | 11091            |                           |                   |
|      |                  | 2384             |                           |                   |
|      |                  | 2218             |                           |                   |
|      |                  | 166              |                           |                   |
|      |                  | 166              |                           |                   |

*Application of Decimals to Concrete Quantities,*  
pages 134, 135.

(1)    ·09375 *acres.*

$$\begin{array}{r} \text{·09375} \\ \underline{\phantom{0}4} \\ \text{·37500} \text{ roods.} \\ \underline{\phantom{0}40} \\ \text{15·00000} \text{ poles.} \end{array}$$

∴ ·09375 *acres* = 15 *poles*.

(2)    3·6285 *degrees.*

$$\begin{array}{r} \text{3·6285} \\ \underline{\phantom{0}60} \\ \text{37·7100} \text{ minutes.} \\ \underline{\phantom{0}60} \\ \text{42·60} \text{ seconds} = 42\frac{3}{5}. \end{array}$$

∴ 3·6285\* = 3° 37' 42 $\frac{3}{5}$ ".

(3)    ·4625 *tons.*

$$\begin{array}{r} \text{·4625} \\ \underline{\phantom{0}20} \\ \text{9·2500} \text{ cwt.} \\ \underline{\phantom{0}4} \\ \text{1·00} \text{ qrs.} \end{array}$$

∴ ·4625 *tons* = 9 *cwt.* 1 *qr.*

(4)    ·4375 *shillings.*

$$\begin{array}{r} \text{·4375} \\ \underline{\phantom{0}12} \\ \text{5·2500} \text{ pence.} \\ \underline{\phantom{0}4} \\ \text{1·00} \text{ farthings.} \end{array}$$

∴ ·4375*s.* = 5 $\frac{1}{4}$ *d.*

\* When the denominations of concrete integral quantities are indicated by small marks or initial letters placed at the upper corner of the units-figure, it is customary to keep the mark or letter in that place, though decimals of the concrete integer should follow, thus: as 3 degrees would be written 3°, so 3·6285 degrees is written 3°·6285; in like manner, 2·413 hours is written 2<sup>h</sup>·413; 5·32 seconds, 5<sup>"</sup>·32; and so on: and even ·6285 degrees, or hours, or minutes, &c., would be written 0°·6285, 0<sup>h</sup>·6285, 0<sup>'</sup>·6285 or 0<sup>m</sup>·6285, according as the minutes refer to 60ths of a degree or to 60ths of an hour.

$$\begin{array}{r}
 (5) \quad 2) \cdot 73125 \times 2\frac{1}{2} \\
 \underline{2} \\
 1\cdot46250 \\
 \cdot 365625 \\
 \hline
 1\cdot828125 \text{ £.} \\
 \underline{20} \\
 16\cdot5625s. \\
 \underline{12} \\
 6\cdot75d. \\
 \underline{4} \\
 3f.
 \end{array}$$

$\therefore$  the value of the decimal is £1 16s. 6 $\frac{3}{4}$ d.

$$\begin{array}{r}
 (6) \quad \cdot 4694 \text{ lbs. troy.} \\
 \underline{12} \\
 5\cdot6328 \text{ oz.} \\
 \underline{20} \\
 12\cdot656 \text{ dwt.} \\
 \underline{24} \\
 2624 \\
 \underline{1312} \\
 15\cdot744 \text{ grs.}
 \end{array}$$

$$\begin{array}{r}
 (7) \quad 12) 3\cdot25d.* \\
 20) 17\cdot2708 \\
 \hline
 19\cdot86354 \text{ £}
 \end{array}$$

$\therefore$  £19 17s. 3 $\frac{1}{4}$ d. = £19·86354.

$\therefore$  ·4694 lbs. troy = 5 oz. 12 dwt. 15·744 grs.

$$(8) \quad \frac{14}{191} \times \frac{21}{751} = \frac{7}{191} \times \frac{21}{379} = \frac{147}{191 \times 379} \quad (9) \quad \begin{array}{r} 12) 9d. \\ 20) \cdot 75s. \\ \hline \cdot 0375 \text{ £} \end{array}$$

$$\begin{array}{r}
 379 \\
 \underline{191} \\
 379 \\
 \underline{3411} \\
 379
 \end{array}$$

$\therefore$  9d. = £·0375.

$$\begin{array}{r}
 72389) 147\cdot000(\cdot 0020307 \\
 \underline{144778} \\
 2222 \\
 \underline{217167} \\
 5033 \\
 \underline{506723}
 \end{array}$$

$$\begin{array}{r}
 (10) \quad 60) 49\cdot7 \\
 60) 48\cdot828333 \\
 \underline{4) 5\cdot8138055} \\
 6) 1\cdot4534514 \\
 \hline
 \cdot 2422419
 \end{array}$$

$\therefore$  5<sup>h</sup> 48<sup>m</sup> 49<sup>s</sup>·7 = 0<sup>d</sup>·2422419

$$\therefore \frac{14}{191} \times \frac{21}{758} = \cdot 0020307$$

\* When farthings are to be reduced to the decimal of a penny it would be puerile to formally divide by 4, because everybody, with

(11)

$$\begin{array}{r}
 2 \text{ lbs. } 7 \text{ oz. } 15 \text{ dwt.} \\
 12 \\
 \hline
 31 \\
 20 \\
 \hline
 635 \text{ dwt.} \\
 \cdot 315 \\
 \hline
 3175 \\
 635 \\
 \hline
 1905 \\
 20) \underline{200\cdot025} \text{ dwt.} \\
 10 \text{ oz. } 0\cdot25 \text{ dwt.} \\
 \underline{24} \\
 100 \\
 \underline{50} \\
 \cdot 600 \text{ gr.} = \frac{3}{4} \text{ gr.}
 \end{array}$$

$\therefore \cdot 315$  of 2 lbs. 7 oz. 15 dwt. =  
10 oz. 0 dwt.  $0\frac{3}{4}$  gr.

sufficient (see Note, p. 126, *Arith.*) It would therefore have been proper here to have used contracted

(12)

$$\begin{array}{r}
 20) \underline{4\cdot5} \text{ s.} \quad 1\cdot46875 \\
 \underline{\text{£}3\cdot225} \quad \underline{3\cdot225} \\
 \quad \quad 734375 \\
 \quad \quad \underline{293750} \\
 \quad \quad 293750 \\
 \quad \quad \underline{40625} \\
 \quad \text{£}4\cdot73671875 \\
 \quad \quad \underline{20} \\
 \quad 14\cdot73437500 \\
 \quad \quad \underline{12} \quad \therefore \text{the product} \\
 \quad \quad 8\cdot812500 \quad \text{is} \\
 \quad \quad \underline{4} \quad \text{£}4 \text{ } 14\text{s. } 8\frac{3}{4}\text{d.} \\
 \quad \quad 3\cdot2500 = 3\frac{1}{4}\text{f.}
 \end{array}$$

In this 12th Example it is plain that an unnecessary number of decimals has been computed; the product to four places would have been amply sufficient. It would therefore have been proper here to have used contracted multiplication, as follows:—

$$\begin{array}{r}
 1\cdot46875 \quad 4\cdot7368 \\
 \underline{5223} \quad \underline{20} \\
 44063 \quad 14\cdot7360 \\
 \underline{2938} \quad \underline{12} \\
 294 \quad 8\cdot832 \\
 \underline{73} \quad \underline{4} \\
 \text{£}4\cdot7368^* \quad \underline{3\cdot328}
 \end{array}$$

$\therefore \text{Prod.} = \text{£}4 \text{ } 14\text{s. } 8\frac{3}{4}\text{d.}$

But the example may be worked very readily by Practice as follows:—

$$\begin{array}{r}
 1\cdot46875 \\
 \underline{3} \\
 4\cdot40625 \\
 4\text{s. } \left| \begin{array}{l} \frac{1}{5} \\ \frac{1}{8} \end{array} \right| \begin{array}{l} 29375 \\ 3672 \end{array} \\
 \hline
 \text{£}4\cdot73672
 \end{array}$$

and the remainder of the operation as above.

the slightest knowledge of decimals, knows that  $\frac{1}{4} = \cdot 25$ ,  $\frac{1}{5} = \cdot 5$ , and  $\frac{3}{4} = \cdot 75$ ; indeed, in Example 9, that follows, 9d. being  $\frac{3}{4}$  of a shilling,  $\cdot 75$  might have been written down at once, without the formality of dividing by 12.

\* It is pretty obvious how it happens that this result, to four decimals, differs from the preceding by a unit in the fourth decimal. It so happens that in all the partial products, except the final one, by 5, the carryings have uniformly increased those products; even in the first step, 2 has been carried, although it is plain that 2 has no more claim to preference than 1. A cautious computer, seeing that all the carryings,

(13) 20)11·25s.

Or thus:

$$\begin{array}{r}
 29 \cdot 25 \overline{) 10 \cdot 5625} \quad ( \cdot 36111 \dots \\
 \underline{8775} \qquad \qquad 20 \\
 17875 \quad 7 \cdot 22222 \dots \\
 \underline{17550} \qquad \qquad 12 \\
 325 \quad 2 \cdot 6666 \dots \\
 \underline{2925} \qquad \qquad 4 \\
 325 \quad 2 \cdot 666 \dots 2\frac{2}{3}
 \end{array}$$

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 10 \quad 11 \quad 3
 \end{array}$$

$$\begin{array}{r}
 20 \\
 \hline
 211 \\
 \hline
 12
 \end{array}$$

$$\begin{array}{r}
 29 \cdot 25 \overline{) 2535} (86 \cdot 66 \dots
 \end{array}$$

$$\begin{array}{r}
 23400 \\
 \hline
 1950 \\
 \hline
 17550 \\
 \hline
 1950
 \end{array}$$

$$\begin{array}{r}
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 \end{array}$$

$\therefore$  the quotient is 7s.  $2\frac{1}{2}d.$  +  $\frac{2}{3}f.$

$$(14) \quad 1 \cdot 36 = \frac{136}{100} = \frac{34}{25}, \text{ and } \cdot 1634 = \frac{1634}{10000} = \frac{817}{5000}.$$

$$\begin{array}{r}
 \text{days.} \quad \text{h.} \quad \text{m.} \quad \text{sec.} \\
 (15) \quad 0 \cdot 5305887 = 12 \quad 44 \quad 2 \cdot 86368 \\
 \underline{24} \\
 21223548 \\
 \underline{10611774} \\
 12 \cdot 7341288 \\
 \underline{60} \\
 44 \cdot 0477280 \\
 \underline{60} \\
 2 \cdot 863680
 \end{array}$$

$$\begin{array}{r}
 (16) \quad 3 \cdot 1416 \overline{) 24857} (7912 \cdot 3 \dagger \\
 \dots \quad 219912 \\
 \underline{28658} \\
 28274 \\
 \underline{384} \\
 314 \\
 \underline{70} \\
 63 \\
 \underline{7}
 \end{array}$$

except in the last partial product, have thus unduly favoured the *increase* of the total result, would feel it prudent to *diminish* that result by unit in the final decimal, if so small a change were of any consequence, which, however, in the present case, it is not.

\* The series of units might evidently be continued to any extent, so that the product below would have an endless series of 2's, the next of 6's, and the next likewise of 6's. And it is plain that  $\frac{2}{3} = \cdot 666 \dots$ , also.

† 3 is put here in preference to 2, solely because it is stated that the final decimal of the divisor is a little *too great*.



$$\begin{array}{r}
 (17) \quad 883220 \text{ miles.} \\
 \underline{61413} \\
 264966 \\
 \underline{8832} \\
 3533 \\
 \underline{88} \\
 53 \\
 \underline{27747.00 \text{ miles.}}
 \end{array}$$

$$\begin{array}{r}
 (18) \quad 12)6.75d. \\
 \underline{20)17.5625s.} \\
 \cdot 878125\text{£} \\
 \cdot 121875\text{£} \\
 \underline{1.000000\text{£, the sum.}}
 \end{array}$$

$$\begin{array}{r}
 \text{Or thus:} \\
 \text{£} \\
 \cdot 121875 \\
 \underline{20} \\
 2.437500s. \\
 \underline{12} \\
 5.2500d. \\
 \underline{4} \\
 1.00f.
 \end{array}$$

$$\begin{array}{l}
 \therefore \text{£} \cdot 121875 = 2s. 5\frac{1}{2}d. \\
 \text{add } 17s. 6\frac{1}{2}d. \\
 \text{the sum } \underline{20s. 0d.}
 \end{array}$$

$$\begin{array}{r}
 (19) \quad \text{£} \\
 \cdot 875 \\
 \underline{20} \\
 17.500s. = 17s. 6d. \\
 \cdot 37cr. \\
 \underline{5} \\
 1.85s. = 1s. 10\frac{1}{2}d. \\
 \underline{12} \\
 10.20d. \quad 19s. 4\frac{1}{2}d., \text{ the sum.}
 \end{array}$$

$$\begin{array}{r}
 (20) \\
 \text{grs. cwt. gr. lbs.} \quad \text{£} \quad \text{£} \quad \text{£ s. d.} \\
 2.5 : 1 \quad 3 \quad 21 :: 1\frac{7}{8} = 1.875 : 5 \quad 16 \quad 3 \\
 \underline{28} \quad \underline{4} \\
 200 \quad \underline{7} \\
 50 \quad \underline{28} \\
 70.0 \quad \underline{217} \\
 70)406.875 \\
 \underline{5.8125} \\
 20 \\
 16.2500 \\
 \underline{12} \\
 3.00
 \end{array}$$

$$\begin{array}{r}
 (21) \quad \text{per.} \\
 40)39 \\
 4)3.975 \\
 24.99375 \text{ ac.} = 24 \text{ ac. } 3 \text{ ro. } 39 \text{ per.}
 \end{array}$$

$$\begin{array}{r}
 \text{h.} \quad \text{h.} \quad \text{ac.} \quad \text{ac. ro. per.} \\
 12.5 : 15.8 :: 24.99375 : 31 \quad 2 \quad 14.736 \\
 \text{or, } 2.5 : \quad :: 4.99875 \\
 \quad .5 : \quad :: .99975 \\
 \quad .1 \quad \quad :: .19995 \\
 \quad \quad \quad 15.8 \\
 \quad \quad \quad 159960 \\
 \quad \quad \quad 99975 \\
 \quad \quad \quad 19995 \\
 \quad \quad \quad \underline{.1)3.159210} \\
 \quad \quad \quad 31.5921 \\
 \quad \quad \quad \underline{4} \\
 \quad \quad \quad 2.3684 \\
 \quad \quad \quad \underline{40} \\
 \quad \quad \quad 14.7360
 \end{array}$$

(22) £6 13s. = £6.65; this must be the third term of the compound proportion.

$$\begin{array}{l} 8 : 20 \\ 3.25 : 9.25 \end{array} \left. \vphantom{\begin{array}{l} 8 : 20 \\ 3.25 : 9.25 \end{array}} \right\} :: £6.65 : \frac{9.25 \times 20}{3.25 \times 8} \quad £6.65 : £47 \text{ 6s. 4.154d.}$$

The fraction is the same as  $\frac{1.85 \times 5}{.65 \times 2} = \frac{.37 \times 5}{.13 \times 2}$

$$\begin{array}{r} £ \\ 6.65 \\ .37 \\ \hline 4655 \\ 1995 \\ \hline 2.4605 \\ 5 \end{array}$$

Instead of multiplying by .37 and 5, and then dividing by .26, we may multiply by 370, and divide by 52.

$$\begin{array}{r} .26)12.3025 \quad (47.31731 \text{ £} \\ \underline{104} \qquad \qquad \qquad 20 \\ 190 \qquad \qquad \qquad 6.3462 \text{ s.} \\ \underline{182} \qquad \qquad \qquad 12 \\ 82 \qquad \qquad \qquad 4.154 \text{ d.} \\ \underline{78} \\ 45 \\ 26 \\ \underline{190} \\ 182 \\ \underline{80} \\ 78 \\ \underline{2} \end{array}$$

(23) £2 13s. 1d. = 53s. 1d. = 53.08333... Or thus:  $\begin{array}{r} £ \text{ s. d.} \\ 2 \quad 13 \quad 1 \\ \hline 20 \\ 53 \\ 12 \\ \hline 637 \\ 34 \\ \hline 2548 \\ 1911 \\ \hline 216 \text{ 58} \\ 26 \\ \hline 129948 \\ 43316 \\ \hline 12)56.3108 \end{array}$

$$\begin{array}{r} .34 \\ \hline 21233333 \\ 159249999 \\ \hline 18.048333 \\ .26 \\ \hline 108289999 \\ 360966666 \\ \hline 4.6925666 \\ 12 \\ \hline 8.310799... \end{array}$$

Product  $\underline{4s. 8.3108d.}$

∴ the product is 4s. 8.3108d.

(24)  $7s. 1\frac{1}{2}d. = 7\frac{1}{8}s. = 7.125s.$ ;  $\pounds 1\ 14s. 8\frac{1}{2}d. = 34s. 8.5d. = 34.70833s...$

$$\begin{array}{r}
 \phantom{34.70833} \cdot 47 \\
 \phantom{34.70833} 49875 \\
 \phantom{34.70833} 28500 \\
 \hline
 \phantom{34.70833} 3.34875' \\
 \phantom{34.70833} \phantom{34.70833} \cdot 23 \\
 \hline
 \phantom{34.70833} 1004625 \\
 \phantom{34.70833} 669750 \\
 \hline
 34.70833 \cdot 7702125 \cdot 022191, \text{ the decimal required.} \\
 \dots 6941666 \\
 \hline
 \phantom{34.70833} 760459 \\
 \phantom{34.70833} 694167 \\
 \hline
 \phantom{34.70833} 66292 \\
 \phantom{34.70833} 34708 \\
 \hline
 \phantom{34.70833} 31584 \\
 \phantom{34.70833} 31237 \\
 \hline
 \phantom{34.70833} 347 \\
 \phantom{34.70833} 347 \\
 \hline
 \end{array}$$

*Recurring, or Circulating Decimals, page 138.*

$$\cdot \dot{1}3\dot{5} = \frac{135}{999} = \frac{45}{333} = \frac{5}{37}, \quad 2\cdot\dot{4}1\dot{8} = 2\frac{418-4}{990} = 2\frac{414}{990} = 2\frac{23}{55},$$

$$\begin{array}{r}
 \phantom{2\cdot\dot{4}1\dot{8}} 414)990(2 \\
 \phantom{2\cdot\dot{4}1\dot{8}} \underline{828} \\
 \phantom{2\cdot\dot{4}1\dot{8}} 162)414(2 \\
 \phantom{2\cdot\dot{4}1\dot{8}} \underline{324} \\
 \phantom{2\cdot\dot{4}1\dot{8}} 90)162(2^* \\
 \phantom{2\cdot\dot{4}1\dot{8}} \underline{180} \\
 \phantom{2\cdot\dot{4}1\dot{8}} \text{G.C.M. } 18)90(5 \\
 \phantom{2\cdot\dot{4}1\dot{8}} \underline{90}
 \end{array}$$

$$\cdot 59\dot{2}\dot{5} = \frac{5925-5}{9990} = \frac{592}{999},$$

$$\cdot 0044\dot{9} = \frac{449-4}{99000} = \frac{445}{99000} = \frac{89}{19800},$$

\* See the improved method for the G. C. M. at page 61.

104 KEY TO THE RUDIMENTARY TREATISE ON ARITHMETIC.

$$3\dot{7}569 = 3\frac{7569}{9999} = 3\frac{841}{1111},$$

$$621\dot{6}21 = 621\frac{621}{999} = 621\frac{207}{333} = 621\frac{23}{37},$$

$$\dot{0}2439 = \frac{2439}{99999} = \frac{1}{41} \quad \text{✎}$$

$$\begin{array}{r} \text{G. C. M. } 2439)99999(41 \\ \underline{9756} \\ 2439 \\ \underline{2439} \end{array}$$

$$\dot{8}57142 = \frac{857142}{999999} = \frac{6}{7} \quad \text{✎}$$

$$\begin{array}{r} 857142)999999(1 \\ \underline{857142} \\ \text{G. C. M. } 142857)857142(6 \\ \underline{857142} \end{array}$$

$$1\dot{0}378 = 1\frac{378}{9999} = 1\frac{42}{1111},$$

$$\dot{0}08497133 = \frac{8497133-8}{999999000} = \frac{8497125}{999999000} = \frac{83}{9768}$$

$$8497125)999999000(118$$

$$\underline{8497125}$$

$$15028650$$

$$\underline{8497125}$$

$$65315250$$

$$\underline{67977000}$$

(See page 61)

$$2661750)8497125(3$$

$$\underline{7985250}$$

$$511875)2661750(5$$

$$\underline{2559375}$$

$$\text{G. C. M. } 102375)511875(5$$

$$\underline{511875}$$

*Extraction of the Square Root, page 148.*

(1) The work of this Example, without any contraction, stands as below; the contracted process is placed beside it.

$$\begin{array}{r}
 31\cdot782153(5\cdot63756623 \\
 \underline{25} \\
 106) \begin{array}{r} 678 \\ 636 \\ \hline \end{array} \\
 1123) \begin{array}{r} 4221 \\ 3369 \\ \hline \end{array} \\
 11267) \begin{array}{r} 85253 \\ 78869 \\ \hline \end{array} \\
 112745) \begin{array}{r} 6384 \\ 563725 \\ \hline \end{array} \\
 1127506) \begin{array}{r} 74675 \\ 6765036 \\ \hline \end{array} \\
 11275126) \begin{array}{r} 702464 \\ 67650756 \\ \hline \end{array} \\
 112751322) \begin{array}{r} 2595644 \\ 225502644 \\ \hline \end{array} \\
 1127513243) \begin{array}{r} 34061756 \\ 3382539729 \\ \hline \end{array} \\
 \underline{\underline{23635871}}
 \end{array}$$

$$\begin{array}{r}
 31\cdot782153(5\cdot6375663 \\
 \underline{25} \\
 106) \begin{array}{r} 678 \\ 636 \\ \hline \end{array} \\
 1123) \begin{array}{r} 4221 \\ 3369 \\ \hline \end{array} \\
 11267) \begin{array}{r} 85253 \\ 78869 \\ \hline \end{array} \\
 1,1,2,7,4) \begin{array}{r} 6384 \\ 5637 \\ \hline \end{array} \\
 \begin{array}{r} 747 \\ 676 \\ \hline \end{array} \\
 \begin{array}{r} 71 \\ 68 \\ \hline \end{array} \\
 \begin{array}{r} 3 \\ 3 \\ \hline \end{array}
 \end{array}$$

The number  $31\cdot782153$ , of which the square root is here found, is confessedly inaccurate in its final decimal; either additional decimals have been suppressed, and no allowance made for them, or else they have been more than compensated for by the addition of a unit to the sixth decimal. If we had any means of knowing that the final 3 above errs a little in defect—that is, that the dismissed decimals have been uncompensated for,—then we should be justified in considering the final 3 in the root, found by the contracted method, as true to the nearest unit: but if on the contrary we knew that the sixth decimal in the proposed number had been increased, then we should consider 2 to have greater claim than 3 as the last figure of the root. Suppose for instance the original number had been  $31\cdot7821533421$ , &c., then, as a glance at the uncontracted method shows, the root to eight decimals would have been  $5\cdot63756626$ ; and therefore the root to seven places,

true to the nearest unit, would have been 5·6375663. But if, on the other hand, the original number had been 31·7821527634, &c., then the root, to the nearest unit in the seventh decimal, would have been 5·6375662. In general, therefore, we cannot be sure that the final decimal in the root is not a unit too great or a unit too little, in the absence of all information as to the nature of the error with which the number proposed for extraction is affected.

$$(2) \quad 1,15 \cdot 297356 (10 \cdot 7376607$$

$$\begin{array}{r} 1 \\ \hline 207) 1529 \\ \quad 1449 \\ \hline 2143) 8073 \\ \quad \quad 6429 \\ \hline 21467) 164456 \\ \quad \quad 150269 \\ \hline 2,1,47,4) 14187 \\ \quad \quad 12884 \\ \hline \quad \quad 1303 \\ \quad \quad 1288 \\ \hline \quad \quad 15 \\ \quad \quad 15 \\ \hline \end{array}$$

$$(3) \quad \cdot 3236068 (5688645$$

$$\begin{array}{r} 25 \\ \hline 106) 736 \\ \quad \quad 636 \\ \hline 1128) 10006 \\ \quad \quad \quad 9024 \\ \hline 1,1,3,6,8) 9828 \\ \quad \quad \quad 9095 \\ \hline \quad \quad \quad 733 \\ \quad \quad \quad 682 \\ \hline \quad \quad \quad 51 \\ \quad \quad \quad 45 \\ \hline \quad \quad \quad 6 \\ \quad \quad \quad 6 \\ \hline \end{array}$$

$$(4) \quad 11(3 \cdot 316625$$

$$\begin{array}{r} 9 \\ \hline 63) 2 \\ \quad 189 \\ \hline 661) 11 \\ \quad \quad 661 \\ \hline 6626) 439 \\ \quad \quad 39756 \\ \hline 6,6,3,2) 4144 \\ \quad \quad 3979 \\ \hline \quad \quad 165 \\ \quad \quad 133 \\ \hline \quad \quad 32 \\ \quad \quad 33 \\ \hline \end{array}$$

$$(5) \quad 47,32,56 (687 \cdot 936$$

$$\begin{array}{r} 36 \\ \hline 128) 1132 \\ \quad \quad 1024 \\ \hline 1367) 10856 \\ \quad \quad 9569 \\ \hline 1,3,7,4) 1287 \\ \quad \quad 1237 \\ \hline \quad \quad 50 \\ \quad \quad 41 \\ \hline \quad \quad 9 \\ \quad \quad 8 \\ \hline \quad \quad 1 \\ \hline \end{array}$$

(6)  $3(1.73205081$

$$\begin{array}{r}
 1 \\
 27 \overline{) 2} \\
 \underline{189} \\
 343 \overline{) 11} \\
 \underline{1029} \\
 3462 \overline{) 71} \\
 \underline{6924} \\
 34640.5 \overline{) 176} \\
 \underline{1732025} \\
 27975 \\
 27712 \\
 \underline{263}
 \end{array}$$

(7)  $90,36,87,89,06,25(950625$

$$\begin{array}{r}
 81 \\
 185 \overline{) 936} \\
 \underline{925} \\
 19006 \overline{) 118789} \\
 \underline{114036} \\
 190122 \overline{) 475306} \\
 \underline{380244} \\
 1901245 \overline{) 9506225} \\
 \underline{9506225}
 \end{array}$$

(8)  $3.65(19.10497317$

$$\begin{array}{r}
 1 \\
 29 \overline{) 265} \\
 \underline{261} \\
 381 \overline{) 4} \\
 \underline{381} \\
 38204 \overline{) 19} \\
 \underline{152816} \\
 382089 \overline{) 37184} \\
 \underline{3438801} \\
 382098 \overline{) 279599} \\
 \underline{267469} \\
 12130 \\
 11463 \\
 \underline{667} \\
 382 \\
 285 \\
 267 \\
 \underline{18}
 \end{array}$$

(9)  $32.398864(5.692$

$$\begin{array}{r}
 25 \\
 106 \overline{) 739} \\
 \underline{636} \\
 1129 \overline{) 10388} \\
 \underline{10161} \\
 11382 \overline{) 22764} \\
 \underline{22764}
 \end{array}$$

(10)  $.00,07,29(.027$

$$\begin{array}{r}
 4 \\
 47 \overline{) 329} \\
 \underline{329}
 \end{array}$$

$7.84.375(28.0067$

$$\begin{array}{r}
 4 \\
 48 \overline{) 384} \\
 \underline{384} \\
 5,6,00 \overline{) 375} \\
 \underline{336} \\
 39 \\
 39 \\
 \underline{\quad}
 \end{array}$$

(12)  $79.182(8.8985$

$$\begin{array}{r}
 64 \\
 168 \overline{) 1518} \\
 \underline{1344} \\
 1,7,69 \overline{) 1742} \\
 \underline{1592} \\
 150 \\
 141 \\
 \underline{\quad} \\
 9 \\
 9 \\
 \underline{\quad}
 \end{array}$$

(13)  $68\cdot736(8\cdot290717701$

$$\begin{array}{r}
 64 \\
 162 \overline{) 473} \\
 \underline{324} \\
 1649 \overline{) 1496} \\
 \underline{14841} \\
 165807 \overline{) 119} \\
 \underline{1160649} \\
 165,814,1 \overline{) 29351} \\
 \underline{165814} \\
 127696 \\
 \underline{116070} \\
 11626 \\
 \underline{11607} \\
 19 \\
 17 \\
 \underline{2}
 \end{array}$$

$$(14) \quad \sqrt{29 \frac{52}{81}} = \sqrt{\frac{29 \times 81 + 52}{81}} = \sqrt{\frac{2401}{81}} = \frac{\sqrt{2401}}{9} = \frac{49}{9} = 5\frac{4}{9}$$

$$\begin{array}{r}
 24,01(49 \\
 \underline{16} \\
 89 \overline{) 801} \\
 \underline{801}
 \end{array}$$

$$(15) \quad \sqrt{104\frac{1}{25}} = \sqrt{\frac{2601}{25}} = \frac{\sqrt{2601}}{5} = \frac{51}{5} = 10\frac{1}{5}$$

$$\begin{array}{r}
 26,01(51 \\
 \underline{25} \\
 101 \overline{) 101} \\
 \underline{101}
 \end{array}$$



(16)  $\sqrt{17\frac{1}{2}} = \sqrt{17.375}$

$$\begin{array}{r}
 17.375(4.16833 \\
 \underline{16} \\
 81) 137 \\
 \underline{81} \\
 826) 565 \\
 \underline{4956} \\
 8,32,8) 694 \\
 \underline{6662} \\
 278 \\
 \underline{250} \\
 28 \\
 \underline{25} \\
 3
 \end{array}$$

(17)  $15\frac{5}{8} = 15.625$

$$\begin{array}{r}
 15.625(3.952347 \\
 \underline{9} \\
 69) 662 \\
 \underline{621} \\
 785) 415 \\
 \underline{3925} \\
 7902) 225 \\
 \underline{15804} \\
 7,9,0,48) 6696 \\
 \underline{6324} \\
 372 \\
 \underline{316} \\
 56 \\
 \underline{55} \\
 1
 \end{array}$$

(18)  $794\frac{1}{2} = 794.2$

$$\begin{array}{r}
 794.2(28.18155425 \\
 \underline{4} \\
 48) 394 \\
 \underline{384} \\
 561) 102 \\
 \underline{561} \\
 5628) 459 \\
 \underline{45024} \\
 6361) 876 \\
 \underline{56361} \\
 563625) 31239 \\
 \underline{2818125} \\
 56,3,6,3,0) 305775 \\
 \underline{281815} \\
 23960 \\
 \underline{22545} \\
 1415 \\
 \underline{1127} \\
 288 \\
 \underline{282} \\
 6
 \end{array}$$

(19)  $34.867844(5.9049$

$$\begin{array}{r}
 25 \\
 109) 986 \\
 \underline{981} \\
 11804) 57844 \\
 \underline{47216} \\
 1180,89) 10628 \\
 \underline{10628}
 \end{array}$$

(20)  $7.6531(2.7664333$

$$\begin{array}{r}
 4 \\
 47) 365 \\
 \underline{329} \\
 546) 3631 \\
 \underline{3276} \\
 5526) 35553 \\
 \underline{33156} \\
 55324) 239715 \\
 \underline{221296} \\
 55,3,2,8) 18419 \\
 \underline{16598} \\
 1821 \\
 \underline{1660} \\
 161 \\
 \underline{166}
 \end{array}$$

*Extraction of the Cube Root, page 134.*

|     |   |              |             |                           |
|-----|---|--------------|-------------|---------------------------|
| (1) | 1 | 0            | 0           | 912,673(97 the cube root. |
|     |   | 9            | 81          | 729                       |
|     |   | <u>9</u>     | <u>81</u>   | 183673 (1)                |
|     |   | 9            | 162         | <u>183673</u>             |
|     |   | 18           | 243         |                           |
|     |   | <u>9</u> (1) | <u>1939</u> |                           |
|     |   | 27           | 26239       |                           |
|     |   | <u>7</u>     |             |                           |
|     |   | 277          |             |                           |

|     |   |              |                |                |
|-----|---|--------------|----------------|----------------|
| (2) | 1 | 0            | 0              | 52,734,375(375 |
|     |   | 3            | 9              | 27             |
|     |   | <u>3</u>     | <u>9</u>       | 25734 (1)      |
|     |   | 3            | 18             | <u>23653</u>   |
|     |   | 6            | 27             | 2081375 (2)    |
|     |   | <u>3</u> (1) | <u>679</u>     | <u>2081375</u> |
|     |   | 9            | 3379           |                |
|     |   | <u>7</u>     | <u>728</u> (2) |                |
|     |   | 97           | 4107           |                |
|     |   | <u>7</u>     | <u>5575</u>    |                |
|     |   | 104          | 416275         |                |
|     |   | <u>7</u> (2) |                |                |
|     |   | 111          |                |                |
|     |   | <u>5</u>     |                |                |
|     |   | 1115         |                |                |

|     |   |              |                 |                |
|-----|---|--------------|-----------------|----------------|
| (3) | 1 | 0            | 0               | 21,024,576(276 |
|     |   | 2            | 4               | 8              |
|     |   | <u>2</u>     | <u>4</u>        | 13024 (1)      |
|     |   | 2            | 8               | <u>11683</u>   |
|     |   | <u>4</u>     | <u>12</u> (1)   | 1341576 (2)    |
|     |   | 2            | 469             | <u>1341576</u> |
|     |   | <u>6</u> (1) | <u>1669</u>     |                |
|     |   | 7            | 518             |                |
|     |   | <u>67</u>    | <u>2187</u> (2) |                |
|     |   | <u>7</u>     | <u>4896</u>     |                |
|     |   | 74           | 223596          |                |
|     |   | <u>7</u> (2) |                 |                |
|     |   | 81           |                 |                |
|     |   | <u>6</u>     |                 |                |
|     |   | 816          |                 |                |

|     |   |              |                 |                     |
|-----|---|--------------|-----------------|---------------------|
| (4) | 1 | 0            | 0               | 80,677,568,161(4321 |
|     |   | <u>4</u>     | <u>16</u>       | 64                  |
|     |   | 4            | 16              | 16677 (1)           |
|     |   | <u>4</u>     | <u>32</u>       | 15507               |
|     |   | 8            | 48              | 1170568 (2)         |
|     |   | <u>4</u> (1) | <u>369</u>      | 1114568             |
|     |   | 12           | 5169            | 56000161 (3)        |
|     |   | 3            | 378             | 56000161            |
|     |   | <u>123</u>   | <u>5547</u>     |                     |
|     |   | 3            | 2584            |                     |
|     |   | <u>126</u>   | <u>557284</u>   |                     |
|     |   | <u>3</u> (2) | <u>2588</u>     |                     |
|     |   | 129          | 559872          |                     |
|     |   | 2            | 12961           |                     |
|     |   | <u>1292</u>  | <u>56000161</u> |                     |
|     |   | 2            |                 |                     |
|     |   | <u>1294</u>  |                 |                     |
|     |   | 2            |                 |                     |
|     |   | <u>1296</u>  |                 |                     |
|     |   | 1            |                 |                     |
|     |   | <u>12961</u> |                 |                     |

|     |   |              |                  |                      |
|-----|---|--------------|------------------|----------------------|
| (5) | 1 | 0            | 0                | 411,001,037,875(7435 |
|     |   | <u>7</u>     | <u>49</u>        | 343                  |
|     |   | 7            | 49               | 68001 (1)            |
|     |   | <u>7</u>     | <u>98</u>        | 62224                |
|     |   | 14           | 147              | 5777037 (2)          |
|     |   | <u>7</u> (1) | <u>856</u>       | 4948407              |
|     |   | 21           | 15556            | 828630875 (3)        |
|     |   | 4            | 872              | 828630875            |
|     |   | <u>214</u>   | <u>16428</u>     |                      |
|     |   | 4            | 6669             |                      |
|     |   | <u>218</u>   | <u>1649469</u>   |                      |
|     |   | <u>4</u> (2) | <u>6678</u>      |                      |
|     |   | 222          | 1656147          |                      |
|     |   | 3            | 111475           |                      |
|     |   | <u>2223</u>  | <u>165726175</u> |                      |
|     |   | 3            |                  |                      |
|     |   | <u>2226</u>  |                  |                      |
|     |   | <u>3</u> (3) |                  |                      |
|     |   | 2229         |                  |                      |
|     |   | 5            |                  |                      |
|     |   | <u>22295</u> |                  |                      |

|     |   |               |                  |                    |
|-----|---|---------------|------------------|--------------------|
| (6) | 1 | 0             | 0                | 7835·8748(19·86228 |
|     |   | <u>1</u>      | <u>1</u>         | <u>1</u>           |
|     |   | 1             | 1                | 6835 (1)           |
|     |   | <u>1</u>      | <u>2</u> (1)     | <u>5859</u>        |
|     |   | 2             | 3                | 976·874 (2)        |
|     |   | <u>1</u> (1)  | <u>351</u>       | <u>903·392</u>     |
|     |   | 3             | 651              | 73·482 (3)         |
|     |   | <u>9</u>      | <u>432</u> (2)   | <u>70·781</u>      |
|     |   | 39            | 1083             | 2·701              |
|     |   | <u>9</u>      | <u>46·24</u>     | <u>2·367</u>       |
|     |   | 48            | 1129·24          | ·334               |
|     |   | <u>9</u> (2)  | <u>46·88</u> (3) | <u>·237</u>        |
|     |   | 57            | 1176·12          | 97                 |
|     |   | <u>·8</u>     | <u>3·57</u>      | <u>94</u>          |
|     |   | 57·8          | 1179·69          | 3                  |
|     |   | <u>·8</u>     | <u>3·6</u>       | <u>—</u>           |
|     |   | 58·6          | 1183·3           |                    |
|     |   | <u>·8</u> (3) | <u>1</u>         |                    |
|     |   | 59·4          | 11,83·4          |                    |
|     |   | <u>6</u>      |                  |                    |
|     |   | 59·5          |                  |                    |

|     |   |               |                  |                     |
|-----|---|---------------|------------------|---------------------|
| (7) | 1 | 0             | 0                | 115·29736 (4·867132 |
|     |   | <u>4</u>      | <u>16</u>        | <u>64</u>           |
|     |   | 4             | 16               | 51297 (1)           |
|     |   | <u>4</u>      | <u>32</u> (1)    | <u>46592</u>        |
|     |   | 8             | 48               | 470536 (2)          |
|     |   | <u>4</u> (1)  | <u>10·24</u>     | <u>419926</u>       |
|     |   | 12            | 58 24            | 50610 (3)           |
|     |   | <u>·8</u>     | <u>10 88</u> (2) | <u>49673</u>        |
|     |   | 12·8          | 69·12            | 937                 |
|     |   | <u>·8</u>     | <u>·8676</u>     | <u>711</u>          |
|     |   | 13·6          | 69·987,6         | 226                 |
|     |   | <u>·8</u> (2) | <u>·871</u> (3)  | <u>213</u>          |
|     |   | 14·4          | 70·859           | 13                  |
|     |   | <u>6</u>      | <u>102</u>       | <u>14</u>           |
|     |   | 14·46         | 70·96,1          |                     |
|     |   | <u>6</u>      | <u>·10</u>       |                     |
|     |   | 14·5,2        | 71,0,6           |                     |
|     |   | <u>6</u> (3)  |                  |                     |
|     |   | 14,6          |                  |                     |

(8)

|   |          |               |                 |
|---|----------|---------------|-----------------|
| 1 | 0        | 0             | 822,650 (93·7   |
|   | <u>9</u> | <u>81</u>     | <u>729</u>      |
|   | 9        | 81            | 93650 (1)       |
|   | <u>9</u> | <u>162</u>    | <u>75357</u>    |
|   | 18       | 243 (1)       | 18293 (2)       |
|   | <u>9</u> | <u>819</u>    | <u>18299953</u> |
|   | 27 (1)   | 25119         |                 |
|   | <u>3</u> | <u>828</u>    |                 |
|   | 273      | 25947 (2)     |                 |
|   | <u>3</u> | <u>195·79</u> |                 |
|   | 276      | 26142·79      |                 |
|   | <u>3</u> |               |                 |
|   | 279 (2)  |               |                 |
|   | <u>7</u> |               |                 |
|   | 279·7    |               |                 |

(9)

|   |           |            |                   |
|---|-----------|------------|-------------------|
| 1 | 0         | 0          | 78,314·6 (42·7839 |
|   | <u>4</u>  | <u>16</u>  | <u>64</u>         |
|   | 4         | 16         | 14314 (1)         |
|   | <u>4</u>  | <u>32</u>  | <u>10088</u>      |
|   | 8         | 48 (1)     | 42266 (2)         |
|   | <u>4</u>  | <u>244</u> | <u>37667</u>      |
|   | 12 (1)    | 5044       | 4599              |
|   | <u>2</u>  | <u>248</u> | <u>4384</u>       |
|   | 122       | 5292 (2)   | 215               |
|   | <u>2</u>  | <u>89</u>  | <u>165</u>        |
|   | 124       | 5381       | 50                |
|   | <u>2</u>  | <u>89</u>  | <u>49</u>         |
|   | 126 (2)   | 5470       | <u>1</u>          |
|   | <u>·7</u> | <u>10</u>  |                   |
|   | 1,2,7     | 548,0      |                   |
|   |           | <u>1</u>   |                   |
|   |           | 5,4,9      |                   |

|      |   |             |                 |                      |
|------|---|-------------|-----------------|----------------------|
| (10) | 1 | 0           | 0               | 12,345,678 (23·11204 |
|      |   | <u>2</u>    | <u>4</u>        | <u>8</u>             |
|      |   | 2           | 4               | 4345 (1)             |
|      |   | <u>2</u>    | <u>8</u>        | <u>4167</u>          |
|      |   | 4           | 12 (1)          | 178678 (2)           |
|      |   | <u>2</u>    | <u>189</u>      | <u>159391</u>        |
|      |   | 6 (1)       | 1389            | 19287 (3)            |
|      |   | <u>3</u>    | <u>198</u>      | <u>16015</u>         |
|      |   | ·63         | 1587 (2)        | 3272                 |
|      |   | <u>3</u>    | <u>6·91</u>     | <u>3204</u>          |
|      |   | 66          | 1593·91         | 68                   |
|      |   | <u>3</u>    | <u>6·92</u>     | <u>64</u>            |
|      |   | 69 (2)      | 1600·83 (3)     | <u>4</u>             |
|      |   | ·1          | ·69             |                      |
|      |   | <u>69·1</u> | <u>1601·5,2</u> |                      |
|      |   | ·1          | 7               |                      |
|      |   | <u>69·2</u> | <u>16,0,2,2</u> |                      |
|      |   | ·1          |                 |                      |
|      |   | 69 3 (3)    |                 |                      |
|      |   | <u>1</u>    |                 |                      |
|      |   | 6,9,3       |                 |                      |

|      |   |             |                  |                        |
|------|---|-------------|------------------|------------------------|
| (11) | 1 | 0           | 0                | 123,456,789 (4·9793386 |
|      |   | <u>4</u>    | <u>16</u>        | <u>64</u>              |
|      |   | 4           | 16               | 59456 (1)              |
|      |   | <u>4</u>    | <u>32</u>        | <u>53649</u>           |
|      |   | 8           | 48 (1)           | 5807789 (2)            |
|      |   | <u>4</u>    | <u>11·61</u>     | <u>5114473</u>         |
|      |   | 12 (1)      | 59·61            | 693316 (3)             |
|      |   | ·9          | 12 42            | 668133                 |
|      |   | <u>12·9</u> | <u>72·03 (2)</u> | <u>25183</u>           |
|      |   | ·9          | 1·0339           | 22313                  |
|      |   | <u>13·8</u> | <u>73·0639</u>   | <u>2870</u>            |
|      |   | ·9          | 1·0388           | 2231                   |
|      |   | 14·7 (2)    | 74 1027 (3)      | 639                    |
|      |   | <u>7</u>    | <u>·1343</u>     | <u>595</u>             |
|      |   | 14·77       | 74·237,0         | 44                     |
|      |   | <u>7</u>    | <u>134</u>       | <u>45</u>              |
|      |   | 14·84       | 74·371           |                        |
|      |   | <u>7</u>    | <u>4</u>         |                        |
|      |   | 14·91 (3)   | 74,3,7,5         |                        |
|      |   | 9           |                  |                        |
|      |   | 14,92       |                  |                        |

|      |     |            |          |               |
|------|-----|------------|----------|---------------|
| (12) | 1   | 0          | 0        | 9(2·080083823 |
|      |     | <u>2</u>   | <u>4</u> | <u>8</u>      |
|      |     | 2          | 4        | 1             |
|      |     | <u>2</u>   | <u>8</u> | (1)           |
|      |     | 4          | 12       | ·998912 (2)   |
|      |     | <u>2</u>   | ·4864    | 1088          |
|      | (1) | 6          | 12·4864  | 103838        |
|      |     | <u>·08</u> | ·4928    | 4962          |
|      |     | 6·08       | 12·9792  | 3894          |
|      |     | <u>8</u>   | <u>5</u> | 1068          |
|      |     | 6·16       | 12·979·7 | 1038          |
|      |     | <u>8</u>   | <u>5</u> | 30            |
|      | (2) | 6·24       | 12·980   | 26            |
|      |     |            |          | <u>4</u>      |
|      |     |            |          | <u>4</u>      |

~~~~~

*Interest, &c., pages 162, 163.*

	(1)	
	£ s. d.	
2)9826	13 8	
	<u>2½</u>	
	19653 7 4	for 2 per cent.
Add	4913 6 10	for ½ per cent.
	<u>245,66 14</u>	2 for 2½ per cent.
	20	
	13,34	
	<u>12</u>	
	4,10	
∴ the interest is	£245 13s. 4d.	

	(2)	
	£	
2)896		
	<u>2½</u>	
	1792	for 2 years.
Add	448	for ½ year.
	<u>4)2240</u>	for 2½ years.
	3¼	
	6720	for 3 per cent.
Add	560	for ¼ per cent.
	<u>72,80</u>	for 3¼ per cent.
	20	
	16,00	
∴ the interest is	£72 16s.	

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 (3) \quad 4)98 \ 19 \ 6 \\
 \hline
 \phantom{4)98 \ 19} 4 \\
 \hline
 \phantom{4)98 \ 19} 395 \ 18 \ 0 \text{ for 4 per cent.} \\
 \text{Subtract } \phantom{4)98 \ 19} 24 \ 14 \ 10\frac{1}{2} \text{ for } \frac{1}{4} \text{ per cent.} \left. \vphantom{\begin{array}{l} 395 \ 18 \ 0 \\ 24 \ 14 \ 10\frac{1}{2} \end{array}} \right\} \text{for 12 months.} \\
 \hline
 \phantom{4)98 \ 19} 12)371 \ 3 \ 1\frac{1}{2} \text{ for } 3\frac{3}{4} \text{ per cent.} \\
 \text{Subtract } \phantom{4)98 \ 19} 30 \ 18 \ 7 \text{ for 1 month.} \\
 \hline
 \phantom{4)98 \ 19} 3,40 \ 4 \ 6\frac{1}{2} \text{ for 11 months.} \\
 \phantom{4)98 \ 19} \underline{20} \\
 \phantom{4)98 \ 19} 8,04 \\
 \phantom{4)98 \ 19} \underline{12} \\
 \phantom{4)98 \ 19} \phantom{8,04} 54 \\
 \phantom{4)98 \ 19} \phantom{8,04} \underline{4} \\
 \phantom{4)98 \ 19} \phantom{8,04} \phantom{54} 2,18
 \end{array}$$

$\therefore$  the interest is £3 8s. 0½d.

$$(4) \quad \text{£}3204 \ 14\text{s.} = \text{£}3204\frac{14}{20} = \text{£}3204\cdot7$$

See Note p. 160, Arithmetic.

$$\begin{array}{r}
 \text{£} \\
 3204\cdot7 \\
 \phantom{3204\cdot7} 37 \\
 \hline
 224329 \\
 96141 \\
 \hline
 3)118574 \\
 \phantom{3)118574} 39524 \\
 \phantom{3)118574} 3952 \\
 \phantom{3)118574} \underline{395} \\
 \phantom{3)118574} 16\cdot2445 \\
 \phantom{3)118574} \underline{20} \\
 \phantom{3)118574} 4\cdot8900 \\
 \phantom{3)118574} \underline{12} \\
 \phantom{3)118574} 10\cdot68 \\
 \phantom{3)118574} \underline{4} \\
 \phantom{3)118574} 2\cdot72
 \end{array}$$

$\therefore$  the interest is £16 4s. 10½d.



(5) From May 7 till August 12 is 97 days  $\therefore$  the work is as follows :—

£
256
9 = $4\frac{1}{2} \times 2$
<u>2304</u>
97
16128
20736
3)223488
74496
7449
744
<u>306177</u>
20
<u>123540</u>
12
<u>28248</u>
4
<u>32992</u>

$\therefore$  the interest is £3 1s.  $2\frac{3}{4}d$ .

(7) The number of days is 249.

£
120
8 = $4 \times 2$
<u>960</u>
249
<u>14940</u>
2241
3)239040
79680
7968
796
<u>327484</u>
20
<u>54968</u>
12
<u>59616</u>

	£	s.	d.
Principal	120	0	0
Interest	3	5	6
Amount	<u>£123</u>	<u>5</u>	<u>6</u>

£	s.	d.
4)319	0	6
		<u>6</u>

	1914	3	0	for 6 years.
Sub.	79	15	$1\frac{1}{2}$	for $\frac{1}{4}$ year.
4)1834	7	$10\frac{1}{2}$		for $5\frac{3}{4}$ years.
		<u>4</u>		
	7337	11	6	at 4 per cent.
Sub.	458	11	$11\frac{1}{2}$	at $\frac{1}{4}$
	68,78	19	$6\frac{1}{2}$	at $3\frac{3}{4}$
		<u>20</u>		
	15,79			
		<u>12</u>		
	9,54			
		<u>4</u>		
	2,18			

$\therefore$  the interest is £68 15s.  $9\frac{1}{2}d$ .

(8) As £100 produces a yearly interest of £5, therefore

£	£	£	s.	£
5	: 100	::	341	5 : 6825
				<u>100</u>
				5)34125
				0
				<u>£6825</u> the principal.

£
(9) Amount 2280
Principal 2000
<u>Interest 280</u>

The interest of the £2000 for one year is  $£20 \times 3\frac{1}{2} = £70$ ; consequently, if 280 be divided by 70, the quotient must be the number of years, which is therefore 4 years.

(10) The interest for 1 year is £42 3s. 9d.  $\times 12 =$  £506 5s.; and if this be multiplied by 100, and the product divided by the rate per cent., the result will, of course, be £11250. Consequently the rate per cent is

$$£506\ 5s. \times 100 \div £11250 = 50625 \div 11250 = 4\frac{1}{2}.$$

£	£	s.	£	s.	d.	£	s.	d.
(11)	100	:	193	12	::	11	18	6
				11		:	23	1
				11				8 $\frac{1}{4}$
			2129	12				
	10s.	$\left  \begin{smallmatrix} \frac{1}{2} \\ \frac{1}{4} \\ \frac{1}{8} \end{smallmatrix} \right $	96	16				
	5s.		48	8				
	2s. 6d.		24	4				
of 5s., 1s.			9	13	7 $\frac{1}{4}$			
			£23	08	13	7 $\frac{1}{4}$		
			20					
			173					
			12					
			8,83					
			4					
			3,33					

$\therefore$  the interest is £23 1s. 8 $\frac{1}{4}$ d.

Otherwise as follows:—

£	s.	£	£	s.	£
193	12	=	193.6	:	and 11
					18.5
					=
					11.925
					193.6
					71550
					35775
					107325
					11925
					23,08.6800
					20
					1,736
					12
					8,832
					4
					3,328

£23 1s. 8 $\frac{1}{4}$ d., the interest.

(12)

3469857  
3288192

3469857 : 100 :: 181665 : 5·236

34·69857 ) 18166500 ( 5·236 *per cent. nearly.*  
17349285

817215  
6939714

1232436  
10409571

19141789

(13)

20936468  
18664761

18664761 : 100 :: 2271707 : 12·17

18664761 ) 227170700 ( 12·17 *per cent.*  
18664761

40523090  
37929522

3193568  
18664761

13270919  
130653327

2055863

(14)

8175124  
6515794

8175124 ) 165933000 ( 20·3 *per cent. nearly*  
16350248

2430520  
24525372

*Discount, pages 165, 166.*

(1) The three months expire on the 10th of April, therefore the bill becomes due on the 13th of April, so that the interest must be calculated for 94 days.

	Otherwise :
12)8 <i>d.</i>	£   s.   d.
20)11 <i>6s.</i>	1264   11   8
£1264·583	8
8	10116   13   4 × 4
10116·666	10
94	101166   13   4
40466....	9
910499....	910500   0   0
3)950966....	40466   13   4
316989	3)950966   13   4
31698	316989
3169	31698
13·02822	3169
20	13·02822
·5644	20
12	·5644
6·7728	12
	6·7728

	£	s.	d.
<i>Bill</i> . . . . .	1264	11	8
<i>Interest or Discount</i> . . .	13	0	7
<i>Present worth</i> . . .	1251	11	1

(2) The four months expire on the 14th of December, and therefore the bill becomes due on the 17th of December. From the 3rd of October till the 3rd of December is 61 days, and therefore to the 17th it is 75 days, for which time the interest must be calculated.

As in last Example 11s. 8d. = £583, therefore the bill to be discounted is £218·583

$$\begin{array}{r}
 218\cdot583 \\
 \phantom{00}8 \\
 \hline
 1748\cdot606 \\
 \phantom{00}75 \\
 \hline
 8743\ldots \\
 122406\ldots \\
 3)131150 \\
 \phantom{00}43717 \\
 \phantom{000}4971 \\
 \phantom{0000}437 \\
 \hline
 1\cdot79675 \\
 \phantom{00}20 \\
 \hline
 15\cdot9350 \\
 \phantom{00}12 \\
 \hline
 11\cdot220
 \end{array}$$

	£	s.	d.
<i>Bill</i> . . . . .	218	11	8
<i>Discount</i> . . . . .	1	15	11
<i>Present worth</i> . . .	216	15	9

(3) The bill becomes due Nov. 30; and from June 3 to Nov. 30 is 180 days, the time for which the interest must be calculated.

$$\begin{array}{l}
 £568 \ 12s. \ 9d. = £568 \frac{12\cdot75}{20} \\
 = £568\cdot6375
 \end{array}$$

$$\begin{array}{r}
 568\cdot6375 \\
 \phantom{00}180 \\
 \hline
 454910000 \\
 5686375 \\
 3)102354\cdot7500 \\
 \phantom{00}34118 \\
 \phantom{000}3411 \\
 \phantom{0000}341 \\
 \hline
 14\cdot0224 \\
 \phantom{00}20 \\
 \hline
 \cdot448 \\
 \phantom{00}12 \\
 \hline
 5\cdot376
 \end{array}$$

	£	s.	d.
<i>Bill</i> . . . . .	568	12	9
<i>Discount</i> . . . . .	14	0	5
<i>Present worth</i> . . .	554	12	4

(4) The bill becomes due on the 2nd of June; and from the 10th of January till the 2nd of June, is 143 days.

$$£1570 \ 10s. \ 6d. = £1570\cdot525$$

$$\begin{array}{r}
 1570\cdot525 \\
 \phantom{00}7 \\
 \hline
 10993\cdot675 \\
 \phantom{00}143 \\
 \hline
 32981025 \\
 43974700 \\
 10993675 \\
 3)1572095\cdot525 \\
 \phantom{00}524032 \\
 \phantom{000}52403 \\
 \phantom{0000}5240 \\
 \hline
 21\cdot53770 \\
 \phantom{00}20 \\
 \hline
 10\cdot754 \\
 \phantom{00}12 \\
 \hline
 9\cdot048
 \end{array}$$

	£	s.	d.
<i>Bill</i> . . . . .	1570	10	6
<i>Discount</i> . . . . .	21	10	9
<i>Present worth</i> . . .	1548	19	9

(5) From the 3rd of July till the 1st of September is 60 days, the time for which the interest must be calculated.

£	s.
39	5
	60
<hr/>	
3)2355	0
785	
78	
7	
<hr/>	
3225	
20	
<hr/>	
6450	
12	
<hr/>	
540	

	£.	s.	d.
<i>Bill</i> . . .	39	5	0
<i>Discount</i> . .	0	6	5½ *
<i>Present worth</i>	38	18	6

(6) The bill becomes due on the 14th of October; and from the 1st of September to this date is 43 days.

£
150
$\frac{11}{1630} = 5\frac{1}{2} \times 2$
<u>43</u>
4950
<u>660</u>
3)70950
23650
2365
<u>236</u>
97201
20
<u>194402</u>
12
<u>52824</u>

	£	s.	d.
<i>Bill</i> . . .	150	0	0
<i>Discount</i> . .	0	19	5
<i>Present worth</i>	149	0	7

	£	£
(7) 1851. Feb. 2. Due	500 × 35 =	17500
March 9. Paid	80	
	<hr/>	
	420 × 67 =	28140
May 15. Paid	115	
	<hr/>	
	305 × 17 =	5185
June 1. Paid	25	
	<hr/>	
	280 × 74 =	20720
Aug. 14. Paid	280	
	<hr/>	
	3)71545	
	23848	
	2384	
	238	
	<hr/>	
	98015	
	20	
	<hr/>	
	16030	

∴ the interest is £9 16s.

\* The banker who discounts the bill would charge this 5½d. as 6d.

*Brokerage, Commission Insurance, &c., page 169.*

$$\begin{array}{r}
 (1) \quad 2) \pounds 3698 \quad 12 \\
 \quad \quad \quad 3\frac{1}{2} \\
 \hline
 \quad \quad 11095 \quad 16 \\
 \quad \quad 1849 \quad 6 \\
 \hline
 \quad 129,45 \quad 2 \\
 \quad \quad 20 \\
 \hline
 \quad \quad 9,02
 \end{array}$$

$\therefore$  the commission is £129 9s.

$$\begin{array}{l}
 (3) \quad \text{Premium} \quad \quad \pounds 5 \quad 5s. \\
 \text{Policy and Commission} \quad 0 \quad 15
 \end{array}$$

$$\pounds 100 - \pounds 6 = \pounds 94 : \pounds 100 :: \pounds 1880 : \pounds 2000$$

$$94)188000 (\pounds 2000 \text{ amount of insurance.}$$

$$(4) \quad \frac{188000}{188000}$$

$$\pounds 100 - \pounds 5.75 = \pounds 94.25 : \pounds 100 :: \pounds 1938 \text{ } 12s. \text{ } 6d. : \pounds 2056 \text{ } 17s. \text{ } 11d.$$

$$\pounds 1938 \text{ } 12s. = \pounds 1938.625, \text{ and this mult. by } 100 \text{ is } 193862.5$$

$$94.25)193862.5 (\pounds 2056 \text{ } 17s. \text{ } 11d. \text{ amt. of insurance.}$$

$$\begin{array}{r}
 18850 \\
 \hline
 53625 \\
 47125 \\
 \hline
 6500 \\
 56550 \\
 \hline
 8450 \\
 20 \\
 \hline
 16900 \\
 9425 \\
 \hline
 7475 \\
 65975 \\
 \hline
 8775 \\
 12 \\
 \hline
 10.5300
 \end{array}$$

$$\begin{array}{r}
 (5) \quad 4) \pounds 876 \quad 5 \quad 10 \\
 \quad \quad \quad 4 \\
 \hline
 \quad \quad 3505 \quad 3 \quad 4 \\
 \text{Subt. } 219 \quad 1 \quad 5\frac{1}{2} \\
 \hline
 \quad 32,86 \quad 1 \quad 10\frac{1}{2} \\
 \quad \quad 20 \\
 \hline
 \quad 17,21 \\
 \quad \quad 12 \\
 \hline
 \quad 2,62 \\
 \quad \quad 4 \\
 \hline
 \quad 2,50
 \end{array}$$

$\therefore$  Commission is £32 17s. 2½d.

(6)

$$\begin{array}{r}
 \pounds \quad s. \quad d. \\
 372 \quad 7 \quad 4 \\
 4s. \quad \frac{1}{8} \quad \left| \begin{array}{r} 74 \quad 9 \quad 6 \\ 9 \quad 6 \quad 2 \end{array} \right. \\
 6d. \quad \frac{1}{2} \quad \left| \begin{array}{r} 83 \quad 15 \quad 8 \\ 20 \end{array} \right. \\
 \hline
 16,75 \\
 \hline
 9,08
 \end{array}$$

$\therefore$  Brokerage is 16s. 9d.

(7) Here the proposed stock must be increased by  $27\frac{3}{4}$  per cent. of itself to give the purchase money.

	£	s.	
4)	912	14	$\times 28$
		7	
	6388	18	
		4	
	25555	12	
Sub. for $\frac{1}{4}$	228	3	6
	253,27	8	6
		20	
	5,48		
		12	
	5,82		

	£	s.	d.
Stock . . .	912	14	0
Increased by .	253	5	6
Purchase money	1165	19	6

(8)	8)	£	s.	d.
		11675	17	0
		14,69	9	7
		20		
		11,89		
		12		
		10,75		

The Brokerage is £14 11s. 10 $\frac{1}{2}$ d., or £14 11s. 11d., accordingly as a farthing is subtracted or added.

(9)	4)	£	s.	d.
		7391	14	9 $\times 6$
			10	
		73917	7	6
			8	
		591339	0	0
		44350	8	6
		1847	18	8
		$\frac{1}{8} = \frac{1}{2}$ of $\frac{1}{4} =$	923	19 4
		6384,61	6	6
		20		
		12,26		
		12		
		3,18		

$\therefore$  the cost is £6384 12s. 3d.

(10)	Insurance	£	s.	d.	
		2	12	6	per cent.
	Commission	0	10	0	per cent.
		3	2	6	per cent.

Policy duty is charged on part of £100, the same as for £100, so that, in the present example duty is paid for £900; which, at 5s. per cent., is £2 5s.

	£
850	
3	
2550	
2s. 6d.   $\frac{1}{8}$	106 5
	26,56 5
	20
	11,25
	12
	3,00

	£	s.	d.
Insurance and commission . . .	26	11	3
Policy duty . . . . .	2	5	0
Whole expense . . . . .	28	16	3



*Proportional Parts, pages 171, 172.*

$$(1) \quad 500 + 650 + 700 = 1850 : 500 :: £555 : £150, A's \text{ share.}$$

There is no absolute necessity for more than one proportion: A's share being found, we may proceed thus:—

<i>sums.</i>	<i>shares.</i>
£	£
500	150, A.
100	30
50	15
650	195, B.
50	15
700	210, C.

$$\begin{array}{r} \text{£} \\ 150 \} = 555 \\ 195 \} = 345 \\ \hline \text{£}210, C's \text{ share.} \end{array}$$

$$\begin{array}{r} 37 : 10 \quad \underline{10} \\ 37) 5550 ( 150 \text{£} \\ \underline{37} \\ 185 \\ \underline{185} \\ 0 \\ 1850 : 650 :: £555 : £195, B's \text{ share.} \\ 37 : 13 :: \underline{13} \\ 37) 7215 ( 195 \text{£} \\ \underline{37} \\ 351 \\ \underline{333} \\ 185 \\ \underline{185} \end{array}$$

$$(2) \quad 100 : 76 :: 112 : 85\frac{3}{25} \text{ of nitre.}$$

$$\begin{array}{r} 76 \\ \underline{76} \\ 672 \\ \underline{784} \\ 85.12 = 85\frac{12}{100} = 85\frac{3}{25} \end{array}$$

$$100 : 14 :: 112 : 15\frac{17}{25} \text{ of charcoal.}$$

$$\begin{array}{r} 14 \\ \underline{14} \\ 448 \\ \underline{112} \\ 15.68 = 15\frac{68}{100} = 15\frac{17}{25} \end{array}$$

$$\begin{array}{r} \text{lbs.} \quad \text{lbs.} \\ 85\frac{3}{25} \} = 112 \\ 15\frac{17}{25} \} = 100\frac{1}{2} \\ \hline 11\frac{1}{2} \text{ lbs. of sulphur.} \end{array}$$

$$(3) \quad \begin{array}{rcl} & \text{wt. of guinea.} & \text{wt. of alloy.} \\ 12 : 1 :: 5 \text{ dwt. } 9\frac{39}{89} \text{ grs.} & : & 10\frac{79}{89} \text{ grs.} \end{array}$$

$$\begin{array}{r} 24 \\ 12 \overline{) 129\frac{39}{89} \text{ grs.}} \quad (10\frac{79}{89} \text{ grs. alloy.} \\ \underline{120} \\ 9\frac{39}{89} = \frac{819}{89} \end{array}$$

dwt. grs.

5  $9\frac{39}{89}$  weight of guinea.0  $10\frac{79}{89}$  weight of alloy.4  $22\frac{58}{89}$  weight of pure gold.

$$(4) \quad \begin{array}{rcl} 66\text{s.} : 27\frac{1}{2}\text{s.} :: 12 \text{ oz.} : \\ \text{or, } 11 : 27\frac{1}{2} :: 2 \text{ oz.} : 5 \text{ oz.} \end{array}$$

$$\begin{array}{r} 2 \\ 11 \overline{) 55} \end{array}$$

5 oz., weight of £1 7s. 6d.

$$\begin{array}{rcl} 40 : 3 :: 5 \text{ oz.} : \\ \text{or, } 8 : 3 :: 1 \text{ oz.} : 7\frac{1}{2} \text{ dwt.} \end{array}$$

20 dwt.

$$8 \overline{) 60}$$

 $7\frac{1}{2}$  dwt., alloy.5 oz., whole weight.  
 $7\frac{1}{2}$  dwt., alloy.4 oz.  $12\frac{1}{2}$  dwt., pure silver.

$$(5) \quad 100 : 11\cdot1 :: 1000 \text{ oz.} : 111 \text{ oz.}$$

$$\begin{array}{r} 1000 \\ 111,00 \text{ oz., hydrogen.} \\ \underline{1000} \end{array}$$

$$\begin{array}{r} 889 \text{ oz., oxygen.} \end{array}$$

$$(6) \quad \begin{array}{rcl} 120 + 80 + 75 = 275 & : & 120 :: \frac{\pounds}{165} : \frac{\pounds}{72} \\ & 55 & 3 \\ & 5 & 5 \overline{) 360} \\ & & \pounds 72, A. \end{array}$$

$$\begin{array}{rcl} 275 : 80 :: \frac{\pounds}{165} : \frac{\pounds}{48} & & \frac{\pounds}{72} \left. \vphantom{\frac{\pounds}{72}} \right\} = \frac{\pounds}{120} \\ \text{or, } 5 : 3 :: 3 & : & 48 \\ & 5 \overline{) 240} & \\ & \pounds 48, B. & \\ & & \frac{\pounds}{48} \left. \vphantom{\frac{\pounds}{48}} \right\} = \frac{\pounds}{45}, C. \end{array}$$

Since  $120 = 1\frac{1}{2}$  times 80, the share of A. might have been found by multiplying that of B by  $1\frac{1}{2}$ .



$$(8) \quad 140 + 105 + 84 + 70 + 60 = 459$$

$$459 : 140 :: £311 \text{ } 15s. : £95 \text{ } 1s. \text{ } 8\frac{3}{4}d. + \frac{101}{153}$$

£311·75

140

1247000

31175

$$459)43645 (£95 \text{ } 1s. \text{ } 8\frac{3}{4}d. + \frac{101}{153} f. \text{ share of } A.$$

4131

2335

2295

40

20

800

459

341

12

4092

3672

420

4

1680

1377

303

459 = 101153

From the share of A, thus determined, the other shares may be deduced, without proportion, thus:—

sums.	shares.
£	£ s. d.
140	95 1 $8\frac{3}{4} + \frac{101}{153} f.$ , A.
70	47 10 $10\frac{1}{4} + \frac{101}{153} f.$ , D.
35	23 15 5 $+ \frac{101}{153} f.$
105	71 6 $3\frac{1}{2} + \frac{101}{153} f.$ , B.
4	<u>4</u>
7)420	7)285 5 $2\frac{1}{2} + \frac{101}{153} f.$
60	40 15 $0\frac{1}{2} + \frac{101}{153} f.$ , E.

and then the share of C may be found as that of E is found in next page.

$$459 : 105 :: 311·75 : 71 \text{ } 6 \text{ } 3\frac{1}{2} + \frac{114}{153}$$

$$153 : 35 \quad 35$$

155875

93525

$$153)10911·25 (£71 \text{ } 6s. \text{ } 3\frac{1}{2}d. + \frac{114}{153} f., \text{ share of } B.$$

1071

201

153

48·25

20

965

918

47

12

564

459

105

4

420

306

114

$$\begin{array}{rcl} & \text{£} & \text{£} \text{ s. } d. \\ 459 : 84 :: 311.75 : 57 \text{ } 1 \text{ } 0\frac{1}{2} = \frac{80}{153} \\ 153 : 28 & \underline{28} & \end{array}$$

$$\begin{array}{r} 249400 \\ 62350 \\ \hline \end{array}$$

$$153)8729 \text{ (£57 } 1\text{s. } 0\frac{1}{2}d. + \frac{80}{153}f., \text{ share of } C.$$

$$\begin{array}{r} 765 \\ \hline 1079 \\ 1071 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 160 \\ 153 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 84 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 336 \\ 306 \\ \hline \end{array}$$

$$\begin{array}{r} .30 \\ \hline \end{array}$$

$$\begin{array}{rcl} & \text{£} & \text{£} \text{ s. } d. \text{ } f. \\ 459 : 70 :: 311.75 : 47 \text{ } 10 \text{ } 10\frac{1}{4} + \frac{127}{153} \\ & 70 & \end{array}$$

$$459)21822.5 \text{ (£47 } 10\text{s. } 10\frac{1}{4}d. + \frac{127}{153}f., \text{ share of } D.$$

$$\begin{array}{r} 1836 \\ \hline \end{array}$$

$$\begin{array}{r} 3462 \\ \hline \end{array}$$

$$\begin{array}{r} 3213 \\ \hline \end{array}$$

$$\begin{array}{r} 249.5 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ \hline \end{array}$$

$$\begin{array}{r} 4990 \\ \hline \end{array}$$

$$\begin{array}{r} 4590 \\ \hline \end{array}$$

$$\begin{array}{r} 400 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \hline \end{array}$$

$$\begin{array}{r} 4800 \\ \hline \end{array}$$

$$\begin{array}{r} 4590 \\ \hline \end{array}$$

$$\begin{array}{r} 210 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \hline \end{array}$$

$$\begin{array}{r} 840 \\ \hline \end{array}$$

$$\begin{array}{r} 459 \\ \hline \end{array}$$

$$\begin{array}{r} 381 \\ \hline \end{array}$$

$$\begin{array}{r} 459 \\ \hline \end{array}$$

Sub. from  
whole sum.

$$\begin{array}{rcl} & \text{£} & \text{s. } d. \\ 95 & 1 & 8\frac{3}{4} + \frac{101}{153}f. \text{ share of } A. \\ 71 & 6 & 3\frac{1}{2} + \frac{111}{153}f. \text{ " of } B. \\ 57 & 1 & 0\frac{1}{2} + \frac{80}{153}f. \text{ " of } C. \\ 47 & 10 & 10\frac{1}{4} + \frac{127}{153}f. \text{ " of } D. \end{array}$$

$$\begin{array}{rcl} 270 & 19 & 11\frac{1}{2} + \frac{66}{153} \\ 311 & 15 & 0 \end{array}$$

$$\begin{array}{rcl} 40 & 15 & 0\frac{1}{4} + \frac{57}{153}f. \text{ share of } E. \end{array}$$

$$\begin{array}{rcl} 381 & = & 127 \\ 459 & = & 153 \end{array}$$

*Chain Rule, pages 172, 173.*

(1)  $3 \text{ lbs. pepper} = 4 \text{ lbs. mustard.}$

$5 \text{ lbs. mustard} = 12 \text{ lbs. candles.}$

*How many lbs. candles = 20 lbs. pepper?*

$$\frac{4 \times 12 \times 20}{3 \times 5} = 4 \times 4 \times 4 = 64 \text{ lbs.}$$

(2)  $5 \text{ lbs. tea} = 12 \text{ lbs. coffee.}$

$9 \text{ lbs. coffee} = 28 \text{ lbs. sugar.}$

$13 \text{ lbs. sugar} = 18 \text{ lbs. soap.}$

*How many lbs. soap = 7 lbs. tea?*

$$\frac{12 \times 28 \times 18 \times 7}{5 \times 9 \times 13} = \frac{12 \times 28 \times 2 \times 7}{5 \times 13} = \frac{4704}{65} = 72\frac{24}{65} \text{ lbs.}$$

(3)  $\text{£}1 = 420d. \text{ Flemish.}$

$58d. \text{ Flemish} = 1 \text{ crown Ven.}$

$10 \text{ cr. Ven.} = 6 \text{ ducats.}$

$1 \text{ duc.} = 360 \text{ merr.}$

$272 \text{ merr.} = 1 \text{ piastre.}$

*How many piastres = £1000?*

$$\frac{420 \times 6 \times 360 \times 1000}{58 \times 10 \times 272} = \frac{90720000}{15776} = 5750\frac{250}{493}.$$

~~~~~  
*Duodecimals, page 174.*

(1) 
$$\begin{array}{r} \text{ft.} \quad \text{in.} \\ 32 \quad 9 \\ \underline{8 \quad 3} \\ 262 \quad 0 \\ \underline{8 \quad 2 \quad 3} \\ 270 \quad 2 \quad 3 \end{array}$$

(2) 
$$\begin{array}{r} \text{ft.} \quad \text{in.} \\ 17 \quad 9 \\ \underline{20 \quad 6} \\ 355 \quad 0 \\ \underline{8 \quad 10 \quad 6} \\ 363 \quad 10 \quad 6 \end{array}$$

(3) 
$$\begin{array}{r} \text{ft.} \quad \text{in.} \\ 65 \quad 10 \\ \underline{29 \quad 6} \\ 589 \quad 2 \\ \underline{132} \\ 32 \quad 11 \\ \underline{1942 \quad 1} \end{array}$$

(4) 
$$\begin{array}{r} \text{ft.} \quad \text{in.} \\ 97 \quad 9 \\ \underline{16 \quad 6} \\ 584 \quad 0 \\ \underline{98} \\ 48 \quad 10 \quad 6 \\ \underline{1612 \quad 10 \quad 6} \end{array}$$

(5) 
$$\begin{array}{r} \text{ft.} \quad \text{in.} \\ 75 \quad 9 \\ \underline{17 \quad 7} \\ 527 \quad 9 \\ \underline{76} \\ 44 \quad 2 \quad 3 \\ \underline{1331 \quad 11 \quad 3} \end{array}$$

(6) 
$$\begin{array}{r} \text{ft.} \quad \text{in.} \\ 97 \quad 8 \\ \underline{8 \quad 9} \\ 781 \quad 4 \\ \underline{73 \quad 3} \\ 854 \quad 7 \end{array}$$

$$\begin{array}{r}
 \text{(7)} \quad \begin{array}{r} \text{ft.} \quad \text{in.} \\ 59 \quad 6 \\ \hline 3 \quad 11 \\ \hline 178 \quad 6 \\ 54 \quad 6 \quad 6 \\ \hline 233 \quad 0 \quad 6 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{(8)} \quad \begin{array}{r} \text{ft.} \quad \text{in.} \\ 87 \quad 5 \\ \hline 35 \quad 8 \\ \hline 439 \quad 7 \\ 262 \\ \hline 58 \quad 3 \quad 4 \\ \hline 3117 \quad 10 \quad 4 \end{array}
 \end{array}$$

*Miscellaneous Examples, page 175.*

$$\begin{array}{r}
 \text{(1)} \quad \begin{array}{r} \pounds \\ 758)21 \quad (\pounds 02770449 \\ \hline 1516 \\ \hline 584 \\ 5306 \\ \hline 534 \\ 5806 \\ \hline 34 \\ 3032 \\ \hline 648 \end{array}
 \end{array}$$

$$.625s. = \frac{625}{1000}s. = \frac{125}{200}s. = \frac{25}{40}s. = \frac{5}{8}s.$$

$$\text{(2)} \quad \frac{25}{12} + 2\frac{1}{2} = \frac{25}{30} = \frac{5}{6}$$

$$\begin{array}{r}
 \text{(3)} \quad \begin{array}{r} \pounds \\ 2)400 \\ \hline 3\frac{1}{2} \\ \hline 1200 \\ 200 \\ \hline 2)1400 \quad \times \quad 2 \quad = \quad \pounds \\ \hline 2\frac{1}{2} \quad \hline 2800 \quad \hline 700 \quad \hline 25200 \\ 140 \end{array}
 \end{array}$$

$$\begin{array}{l}
 \text{Int. for } 2\frac{1}{2} \text{ years, } \pounds 35.00 \\
 \text{Int. for 59 days, } \pounds 2 \text{ } 5 \text{ } 3 \\
 \text{Int. required, } \pounds 37 \text{ } 5s. \text{ } 3d.
 \end{array}$$

$$\begin{array}{r}
 2800 \\
 59 \\
 \hline 25200 \\
 140 \\
 \hline 3)165200 \\
 55066 \\
 \hline 5506 \\
 550 \\
 \hline 226322 \\
 20 \\
 \hline 52644 \\
 12 \\
 \hline 31728
 \end{array}$$

$$\begin{array}{r}
 \text{(4)} \qquad \qquad \qquad \begin{array}{r}
 \text{£} \qquad \text{s.} \\
 2)712 \qquad 6 \\
 \hline
 \qquad \qquad 7\frac{1}{2} \\
 4986 \qquad 2 \\
 356 \qquad 3 \\
 \hline
 \end{array} \\
 4 \text{ mo.} = \frac{1}{3} \text{ yr.} \quad 3)5342 \qquad 5 \text{ Int. for 1 year.} \\
 \text{Subtract} \quad 1780 \qquad 15 \text{ Int. for 4 months.} \\
 \hline
 3561 \qquad 10 \\
 \qquad \qquad 20 \\
 \hline
 1230 \\
 \qquad \qquad 12 \\
 \hline
 36 \\
 \qquad \qquad 4 \\
 \hline
 24 \\
 \hline
 \end{array}$$

$\therefore$  the interest for 8 months is £35 12s.  $3\frac{1}{2}$ d.

$$\begin{array}{r}
 \text{(5)} \qquad \qquad \qquad 73 \text{ lbs. } 5 \text{ oz.} = 881 \text{ oz., at } 5\text{s. } 9\text{d.} \\
 \qquad \qquad \qquad \begin{array}{r}
 \text{s.} \quad \text{d.} \\
 5 \quad 9 \\
 \hline
 5286 \\
 5 \text{ dwt. } \left| \frac{1}{4} \right| \begin{array}{r} 1 \quad 5\frac{1}{4} \\ \hline 4 \quad 3\frac{3}{4} \end{array} \quad 3\text{d. } \left| \frac{1}{4} \right| \begin{array}{r} 220 \quad 3 \\ \hline 5065\text{s. } 9\text{d., price of 881 oz.} \\ 4 \quad 3\frac{3}{4} \text{ price of 15 dwt.} \end{array} \\
 \hline
 20)5070 \quad 0\frac{3}{4} \\
 \hline
 \text{price of the whole, } \underline{\underline{\text{£}253 \quad 10 \quad 0\frac{3}{4}}}
 \end{array}$$

$$\begin{array}{rcll}
 \text{(6)} & 14 \text{ lbs.} & : & 8 \text{ cwt. } 3 \text{ qrs. } 7 \text{ lbs.} :: 10\text{d.} \\
 & 14 \text{ lbs.} & : & 35 \text{ qrs. } 7 \text{ lbs.} :: 10\text{d.} \quad \text{d.} \quad \text{d.} \\
 & 1 & : & 70\frac{1}{2} :: 10\text{d.} : 70\frac{1}{2} \times 10 = 705 = \\
 & & & 58\text{s. } 9\text{d.} = \text{£}2 \text{ } 18\text{s. } 9\text{d.}
 \end{array}$$

(7) As the answer is to be *men*, the 16 men must be the third term of the compound proportion: the first antecedent and consequent must therefore be 54 : 135, or 6 : 15, or 2 : 5; the second antecedent and consequent must be 8 : 6, or 4 : 3; therefore the stating is

$$\begin{array}{l}
 2 : 5 \\
 4 : 3
 \end{array}
 \left. \vphantom{\begin{array}{l} 2 : 5 \\ 4 : 3 \end{array}} \right\} :: 16 \text{ men} : 30 \text{ men, the number required.}$$

$$\frac{5 \times 3 \times 16}{2 \times 4} = 5 \times 3 \times 2 = 30.$$



(8)

7985 at 7s. 10½d.

$$\begin{array}{r} 7 \\ \hline 55895 \\ 6d. \mid \frac{1}{8} \mid 3992 \quad 6 \\ of \ 1s., \ 4d. \mid \frac{1}{8} \mid 2661 \quad 8 \\ \frac{1}{4}d. \mid \frac{1}{16} \mid 166 \quad 4\frac{1}{2} \\ 20) \underline{6271,5} \quad 6\frac{1}{2} \\ \underline{\pounds 3135 \ 15s. \ 6\frac{1}{2}d.} \end{array}$$

(9)

|    |     |    |    |
|----|-----|----|----|
|    | £   | s. | d. |
| 8) | 530 | 2  | 9  |
|    | 66  | 5  | 4  |
|    | 20  |    |    |
|    | 13  | 25 |    |
|    |     | 12 |    |
|    | 3   | 0  | 4  |

∴ the commission is 13s. 3d.

(10)

|       |         |    |
|-------|---------|----|
|       | £       | s. |
| 4)    | 4563    | 10 |
|       |         | 7  |
|       | 31944   | 10 |
|       |         | 4  |
|       | 127778  | 0  |
| Subt. | 1140    | 15 |
|       | 1266,37 | 5  |
|       | 20      |    |
|       | 7,45    |    |
|       | 12      |    |
|       | 5,40    |    |

|       |    |    |                  |
|-------|----|----|------------------|
| £     | s. | d. |                  |
| 4563  | 10 | 0  | at 100 per cent. |
| 1266  | 7  | 5  | at 27½ per cent. |
| £5829 | 17 | 5  | the price.       |

(11)

|              |             |      |      |      |                      |     |                                   |
|--------------|-------------|------|------|------|----------------------|-----|-----------------------------------|
| cwt.         | qrs.        | lbs. | cwt. | £    | s.                   | d.  |                                   |
| 5            | 3           | 14   | : 1  | : 2  | 10                   | 11, | gain on the whole, : 8s. 8d, gain |
| 4            |             |      | 4    | 20   |                      |     | [per cwt.                         |
| 23           |             |      | 4    | 50   |                      |     |                                   |
| 2            |             |      | 2    | 12   |                      |     |                                   |
| hlf. qrs. 47 | hlf. qrs. 8 |      |      | 611  |                      |     |                                   |
|              |             |      |      | 8    |                      |     |                                   |
|              |             |      | 47)  | 4888 | (104 pence = 8s. 8d. |     |                                   |
|              |             |      |      | 47   |                      |     |                                   |
|              |             |      |      | 188  |                      |     |                                   |
|              |             |      |      | 188  |                      |     |                                   |

$$(12) \quad \begin{array}{r} \text{£} \quad \text{s.} \quad \text{£} \quad \text{s.} \quad \text{d.} \\ 100 : 115 :: 4 \quad 16 : 5 \quad 10 \quad 4\frac{3}{4} \end{array}$$

$$\begin{array}{r} 4s. \quad | \quad \frac{1}{2} \quad | \quad \begin{array}{r} 575 \\ 23 \\ \hline 5,52 \\ 20 \\ \hline 10,40 \\ 12 \\ \hline 4,80 \\ 4 \\ \hline 3,20 \end{array} \end{array}$$

(13) To produce the same amount of interest at the same rate per cent., the principal must evidently increase as the time diminishes; hence the proportion is

$$\begin{array}{r} \text{yrs.} \quad \text{yrs.} \quad \text{£} \quad \text{s.} \quad \text{£} \quad \text{s.} \\ 3\frac{1}{2} : 5\frac{3}{4} :: 210 \quad 3 : 350 \quad 5 \\ \hline 39 : 65 \quad 12609 \quad 0 \\ \text{£}210 \quad 3s. \times 5 = 1050 \quad 15 \quad \text{£} \quad \text{s.} \\ \hline 39 \quad 13659 \quad 15 \quad (350 \quad 5) \\ \hline 117 \\ \hline 195 \\ 195 \\ \hline 9 \\ 20 \\ \hline 195 \\ 195 \\ \hline \end{array}$$

$$\begin{array}{r} (16) \quad \begin{array}{r} 12)26(2\frac{1}{2} \\ \underline{6} \\ 56)166(3 - \frac{3}{4} \\ \underline{14} \quad (\text{See p. 61.}) \\ 624 \\ 156 \\ \hline 182)2184(12 \\ \underline{182} \\ 364 \\ 364 \\ \hline \end{array} \end{array}$$

$\therefore 2184$  is the L. C.M.

(14) Sound moves 1125 feet in one second (Arith., p. 19), and there are 1760 yards in a mile.

$$\begin{array}{r} 1760 \\ \underline{120} \\ 211200 \\ \underline{3} \\ 1125)633600(563\frac{1}{3} \text{ sec.} = 9 \text{ min.} \\ 5625 \quad \underline{23\frac{1}{3} \text{ sec.}} \\ \hline 7110 \\ 6750 \\ \hline 3600 \\ 3375 \\ \hline 225 \\ 225 \\ \hline \end{array}$$

(15) The supply is at the rate of  $\frac{1}{5}$  gals. in a minute, the leakage at the rate of  $\frac{1}{3}$  gal. in a minute; consequently the water accumulates in the cistern at the rate of  $\frac{1}{5} - \frac{1}{3} = \frac{16}{15}$  gals. in a minute; and  $400 \div \frac{16}{15} = \frac{400 \times 15}{16} = \frac{100 \times 15}{4} = \frac{1500}{4} = 375$ , the number of minutes = 6 hours 15 minutes.

(17) The long hand, starting at 9 o'clock from XII., comes up to the short hand, when it has gained 11 times the space the short hand has passed over after starting from IX.; the number of minutes in this space will therefore be found by dividing the 9 hours' gain of the long hand by 11  $\therefore$  the time is  $\frac{99}{11}$  min. = 9 min.  $54\frac{6}{11}$  sec. past 9 o'clock, or 10 min.  $54\frac{6}{11}$  sec. to 10 o'clock.



|      |                                                                                                                             |                                                                                                                                         |                                                                                                                                                                                                                                                               |
|------|-----------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (22) | $\begin{array}{r} 73 \\ 73 \\ \hline 219 \\ 511 \\ \hline 5329 \\ 73 \\ \hline 15987 \\ 37303 \\ \hline 389017 \end{array}$ | $\begin{array}{r} 27 \\ 27 \\ \hline 189 \\ 54 \\ \hline 729 \\ 729 \\ \hline 6561 \\ 1458 \\ \hline 5103 \\ \hline 531441 \end{array}$ | $\begin{array}{r} 57,19,14,06,25(7,56,25(275 \\ 49 \quad 4 \\ \hline 145) 819 \quad 47)356 \\ 725 \quad 329 \\ \hline 1506) 9414 \quad 545) 2725 \\ 9036 \quad 2725 \\ \hline 15122) 37806 \\ 30244 \\ \hline 151245) 756225 \\ 756225 \\ \hline \end{array}$ |
|------|-----------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

$$\therefore 73^3 = 389017; 27^4 = 531441; \sqrt[4]{5719140625} = 275$$

(23)  $\begin{array}{l} \text{oz.} \\ \frac{2}{3} : \frac{5}{6} \times 16 :: \frac{7}{8} : \end{array}$   
 $\begin{array}{l} \frac{2}{3} : \frac{5}{3} \times 8 :: \frac{7}{8} : \end{array}$   
 $2 : 5 \times 8 :: \frac{7}{8} : \frac{5 \times 8 \times 7}{2 \times 8} s. = 17s. 6d.$

(24)  $\begin{array}{l} 111\frac{1}{8} : 100 :: £10000 : £8978 \quad 13s. \quad 6d. \\ 891 : 800 \end{array}$

$$\begin{array}{r} 891 \overline{) 8000000} (8978 \quad 13 \quad 6 \\ \underline{7128} \\ 8720 \\ \underline{8019} \\ 7010 \\ \underline{6237} \\ 7730 \\ \underline{7128} \\ 602 \\ \underline{20} \\ 12040 \\ \underline{891} \\ 3130 \\ \underline{2673} \\ 457 \\ \underline{12} \\ 5484 \\ \underline{5346} \\ 138 \end{array}$$

(25)  $\frac{7}{16 \times 16} lb. = .02734375 lb.$   
 $\begin{array}{r} 8) 7 \\ \hline 8) .875 \\ \hline 4) .109375 \\ \hline .02734375 \end{array}$

(26)

$$\begin{array}{r} \pounds 350 \\ 4 \\ \hline 1400 \\ 8 \\ \hline 11200 \end{array}$$

∴ the banker's discount is £112.

The interest of £100 for 8 years at 4 per cent. is £32  $\therefore$  the *amount* of £100 in 8 years is £132; therefore

$$132 : 100 :: £350 : £265\ 3s.\ 0\frac{1}{2}d.$$
$$132 = 12 \times 11$$
$$\begin{array}{r} 100 \\ 12 \overline{) 35000} \\ 11 \overline{) 2916} \end{array} \quad \begin{array}{l} 8 \\ 1 \end{array} \left. \vphantom{\begin{array}{r} 100 \\ 12 \overline{) 35000} \\ 11 \overline{) 2916} \end{array}} \right\} = \begin{array}{l} \pounds \\ 20 \text{ rem.} \\ 20 \end{array}$$

132) 400(3 047.

|                                |     |    |                  |     |
|--------------------------------|-----|----|------------------|-----|
|                                | £   | s. | d.               | 396 |
|                                | 350 | 0  | 0                | 4   |
| <i>True present worth . .</i>  | 265 | 3  | 0 $\frac{1}{4}$  | 12  |
| <i>True discount . . . . .</i> | 84  | 16 | 11 $\frac{3}{4}$ | 48  |
| <i>Bankers' discount . . .</i> | 112 | 0  | 0                | 4   |
| <i>Difference . . . . .</i>    | 27  | 3  | 0 $\frac{1}{4}$  | 192 |

(27)

(27) £ s. £ s.  
£100—3 8=96 12=£96.6

$$\therefore 96.6 : 100 :: £3500 :$$

13.8

500

100

2 3

13-8) 50000(3623 3 9

414 *the sum reqd.*

860

828

---

320

276

---

44

414

$$\frac{11}{20}$$

2

520

44

 $\frac{1}{14}$ 

1

19

12

1

(28)

Here we shall have to multiply 51.425 by 4.125, and to divide the product by 100: it is obvious that *three* will be decimal places enough to retain in the product.

51.425

5214

905700

205700  
5142

5143  
1028

257

2 10100

2,12128  
90

20

2,4256

---

12

$\therefore$  the interest is £2 2s. 5d.

(29)

The interest must obviously be multiplied by 100, and the result divided by the product of the rate and time, therefore

$$\pounds \frac{35}{1\frac{3}{4} \times 4} = \pounds \frac{35}{7} = \pounds 5, \text{ the sum required.}$$

(30)

|                      | £   | £                  | £         |
|----------------------|-----|--------------------|-----------|
| <i>Aug. 12 due</i>   | 170 | $\times 37 = 6290$ | 3 ) 11334 |
| <i>Sept. 18 paid</i> | 54  |                    | 3778      |
|                      | 116 | $\times 29 = 3364$ | 377       |
| <i>Oct. 17 „</i>     | 56  |                    | 37        |
|                      | 60  | $\times 28 = 1680$ | 1,5526    |
| <i>Nov. 14 „</i>     | 60  | 11334              | 20        |
|                      |     |                    | 11,052    |
|                      |     |                    | 12        |
|                      |     |                    | 624       |
|                      |     |                    | 4         |
|                      |     |                    | 2,496     |

$\therefore$  the interest due is £1 11s. 0½d.

(31)

| £           | s.                                         |
|-------------|--------------------------------------------|
| 3           | 10                                         |
| 20          |                                            |
| 70          |                                            |
| 17          |                                            |
| 1190        |                                            |
| 12          |                                            |
| 14280       |                                            |
| 2           |                                            |
| 4½ × 2 = 9  | 28560                                      |
| 112 ) 3173½ | ( 28 cwt. 37½ lbs. = 28 cwt. 1 qr. 9½ lbs. |
| 224         |                                            |
| 933         |                                            |
| 896         |                                            |
| 37          |                                            |

(32) It appears from the question that 1 gal. rum is equal to  $\frac{7}{9}$  gal. brandy, and that 1 gal. geneva is equal to  $\frac{7}{12}$  gal. brandy: consequently

$$1 \text{ gal. B.} + 1 \text{ gal. R.} + 1 \text{ gal. G.}$$

$$= (1 + \frac{7}{9} + \frac{7}{12}) \text{ gal. B.}$$

that is to say, three gallons, one of each spirit, are worth

$$\frac{36+28+21}{36} \text{ or } \frac{85}{36} \text{ gals. of brandy:}$$

hence to get the price of a gallon of brandy, we have only to divide £2 2s. 6d., that is  $42\frac{1}{2}$ s. by  $\frac{85}{36}$ .

$$\begin{array}{r} 42\frac{1}{2} \\ 36 \overline{) 252} \\ 126 \\ 18 \overline{) 1530} \\ 85 \overline{) 1530} \quad (18s. \text{ Brandy.} \\ 680 \\ 680 \end{array}$$

$$\therefore 18s. \times \frac{7}{9} = 14s. \text{ Rum.}$$

$$18s. \times \frac{7}{12} = 10s. 6d. \text{ Geneva.}$$

(33) The interest per cent. for 9 months is  $\frac{3}{4}$  of £5, that is, £3 $\frac{3}{4}$ , so that the stating is

$$103\frac{3}{4} : 100 :: £357\frac{1}{2} :$$

$$\begin{array}{r} 4 \overline{) 415} \\ 1430 \\ 100 \overline{) 143000} \quad \begin{array}{l} £ \quad s. \quad d. \\ 344 \quad 11 \quad 6\frac{3}{4} \end{array} \\ 1245 \quad \text{the present worth.} \\ 1850 \\ 1660 \overline{) 1900} \\ 1660 \overline{) 240} \\ 20 \overline{) 4800} \\ 4565 \overline{) 235} \\ 12 \overline{) 2820} \\ 2490 \overline{) 330} \\ 4 \overline{) 1320} \\ 1245 \overline{) 75} \end{array}$$

$$(34) \quad \sqrt{27\frac{9}{16}} = \frac{\sqrt{441}}{4}, \text{ and } \sqrt{9\frac{49}{40}} = \frac{\sqrt{484}}{7}.$$

$$\begin{array}{r} 4,41 \quad (21 \\ 4 \overline{) 41} \\ 41 \overline{) 41} \\ 41 \overline{) 21} \\ 21 \overline{) 22} \\ 22 \overline{) 22} \end{array} \quad \begin{array}{r} 4,84 \quad (22 \\ 4 \overline{) 84} \\ 42 \overline{) 84} \\ 84 \overline{) 84} \end{array}$$

$$\frac{21}{4} - \frac{22}{7} = 5\frac{1}{4} - 3\frac{1}{7} = 2\frac{3}{28}, \text{ first answer.}$$

$$\begin{array}{r} 27,04 \quad (52 \\ 25 \overline{) 204} \\ 102 \overline{) 204} \\ 204 \end{array} \quad \begin{array}{r} 23,04 \quad (48 \\ 16 \overline{) 704} \\ 88 \overline{) 704} \\ 704 \end{array}$$

[see over.]

$$\begin{array}{r}
 42,25 \text{ (65} \\
 \underline{36} \\
 125) \text{ } 625 \\
 \underline{625} \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 51,84 \text{ (72} \\
 \underline{49} \\
 142) \text{ } 284 \\
 \underline{284} \\
 \hline
 \end{array}$$

$$\therefore \sqrt{\frac{2704}{4225}} = \frac{52}{65}, \quad \sqrt{\frac{2304}{5184}} = \frac{48}{72} = \frac{2}{3}$$

$$\text{and } \frac{52}{65} - \frac{2}{3} = \frac{156 - 130}{195} = \frac{26}{195} = \frac{2}{15}, \text{ second answer.}$$

(35)

|   |             |                     |                                          |
|---|-------------|---------------------|------------------------------------------|
| 1 | 0           | 0                   | $\cdot 5(793701 = \sqrt[3]{\frac{1}{2}}$ |
|   | $\cdot 7$   | $\cdot 49$          | $\cdot 343$                              |
|   | $\cdot 7$   | $\cdot 49$          | $\cdot 157$                              |
|   | $\cdot 7$   | $\cdot 98$          | $\cdot 150039$                           |
|   | $1\cdot 4$  | $1\cdot 47$         | $6961$                                   |
|   | $\cdot 7$   | $\cdot 1971$        | $5638$                                   |
|   | $2\cdot 1$  | $1\cdot 6671$       | $1323$                                   |
|   | 9           | $\cdot 2052$        | $1321$                                   |
|   | $2\cdot 19$ | $1\cdot 8723$       | $2$                                      |
|   | 9           | 71                  | $2$                                      |
|   | $2\cdot 28$ | $1\cdot 879\cdot 4$ |                                          |
|   | 9           | 7                   |                                          |
|   | $2\cdot 37$ | $1\cdot 886$        |                                          |
|   |             | 17                  |                                          |
|   |             | $1\cdot 8,8,77$     |                                          |

  

|   |               |                  |                                                   |
|---|---------------|------------------|---------------------------------------------------|
| 1 | 0             | 0                | $7\cdot 2(1\cdot 930979 = \sqrt[3]{7\frac{1}{2}}$ |
|   | $\frac{1}{1}$ | $\frac{1}{1}$    | $\frac{1}{62}$                                    |
|   | $\frac{1}{1}$ | $\frac{1}{2}$    | $5859$                                            |
|   | $\frac{1}{2}$ | $\frac{3}{3}$    | $341$                                             |
|   | $\frac{1}{3}$ | $3\cdot 51$      | $330057$                                          |
|   | $\frac{3}{3}$ | $6\cdot 51$      | $10943$                                           |
|   | $\cdot 9$     | $4\cdot 32$      | $10062$                                           |
|   | $3\cdot 9$    | $10\cdot 83$     | $881$                                             |
|   | 9             | $\cdot 1719$     | $782$                                             |
|   | $4\cdot 8$    | $11\cdot 0019$   | $99$                                              |
|   | 9             | $\cdot 1728$     | $100$                                             |
|   | $5\cdot 7$    | $11\cdot 1747$   |                                                   |
|   | 3             | 52               |                                                   |
|   | $5\cdot 73$   | $11\cdot 1,7,99$ |                                                   |
|   | 3             |                  |                                                   |
|   | $5\cdot 76$   |                  |                                                   |
|   | 3             |                  |                                                   |
|   | $5\cdot 79$   |                  |                                                   |





(37) A can do  $\frac{1}{10}$  of the work in a day, and B can do  $\frac{1}{15}$  of it; so that if both work together, they can do  $\frac{1}{10} + \frac{1}{15} = \frac{23}{150}$  of it in one day; consequently they must occupy as many days as there are parts equal to  $\frac{23}{150}$  in the whole, or

$\therefore 1 \div \frac{23}{150} = \frac{150}{23} = 5\frac{15}{23}$ ,  
the number of days required.

(38)

$$854\frac{1}{4} \times 62\frac{1}{2} = \frac{3457}{4} \times \frac{125}{2}$$

$$\begin{array}{r} 3457 \\ 125 \\ \hline 17285 \\ 41484 \\ \hline 8)432125 \\ 9)54015...5 \\ \hline 6001\frac{53}{72} \text{ sq. yds.} \end{array}$$

(39)  $\begin{array}{ccc} \text{£} & \text{s.} & \text{d.} \\ 675 & 11 & 8 \end{array}$  at  $\begin{array}{ccc} \text{£} & \text{s.} & \text{d.} \\ 5 & 13 & 9 \end{array}$  per cent.

$$\begin{array}{r} 10\text{s.} \left| \begin{array}{l} 1\frac{1}{2} \\ 2\text{s. } 6\text{d.} \left| \begin{array}{l} 1\frac{1}{2} \\ 1\frac{1}{2} \end{array} \right. \right. \\ \text{sl. } 3\text{d.} \left| \begin{array}{l} 1\frac{1}{2} \\ 1\frac{1}{2} \end{array} \right. \end{array} \right. \begin{array}{r} 3377 \ 18 \ 4 \\ 337 \ 15 \ 10 \\ 84 \ 8 \ 11\frac{1}{2} \\ 42 \ 4 \ 5\frac{1}{2} \\ \hline 38,42 \ 7 \ 7\frac{1}{4} \\ 20 \\ \hline 8,47 \\ 12 \\ \hline 5,71 \\ 4 \\ \hline 2,85 \end{array}$$

(40)  $\begin{array}{ccc} \text{£} & \text{s.} & \text{d.} \\ 280 \times 5 = 1400, & 266 & 13 \ 4 \times 6 = 1600 \end{array}$

$1400 + 1600 = 3000 : 1400 :: 331 \ 12 \ 6 : 15 : 7$

$\begin{array}{r} \text{£} \quad \text{s.} \quad \text{d.} \\ 3)2321 \ 7 \ 6 \\ \hline 5)773 \ 15 \ 10 \\ \hline A's \ share \ . \ . \ \text{£}154 \ 15 \ 2 \\ \text{whole profits} \ . \ . \ \text{£}331 \ 12 \ 6 \\ \hline B's \ share \ . \ . \ \text{£}176 \ 17 \ 4 \end{array}$

$\therefore$  the premium is £38 8s.  $5\frac{1}{4}$ d.

(41)  $\begin{array}{ccc} \text{£} & \text{s.} & \\ 89 & 5 \times 5 = 446 & 5 \\ 92 & 15 \times 7 = 649 & 5 \\ 38 & 10 \times 11 = 423 & 10 \end{array}$

$\begin{array}{ccc} \text{£} & \text{s.} & \\ 1519 & 0 : 446 \ 5 :: 86 \ 16 : 25 \ 10 \\ & : 446 \cdot 25 :: 86 \cdot 8 \\ & 86 \cdot 8 \end{array}$

$\begin{array}{r} 357000 \\ 267750 \\ \hline 357000 \end{array} \quad \begin{array}{ccc} \text{£} & \text{s.} & \\ 1519)38734 \cdot 500 (25 \cdot 5 = 25 & 10, & A's \ share. \\ 3038 \\ \hline 8354 \\ 7595 \\ \hline 7595 \\ 7595 \end{array}$

(41) *continued.*

|       |              |             |            |                               |
|-------|--------------|-------------|------------|-------------------------------|
|       |              | £           | £          | s.                            |
| 1519  | : 423·5      | :: 86·8     | : 24       | 4                             |
|       | 86·8         |             |            |                               |
|       | <u>33880</u> |             |            |                               |
|       | 25410        |             |            |                               |
|       | <u>33880</u> | £           | £          | s.                            |
| 1519) | 36759·80     | (24·2 =     | 24         | 4, <i>C's share.</i>          |
|       | <u>3038</u>  |             | 25         | 10, <i>A's share.</i>         |
|       | 6379         | <i>Sub.</i> | 49         | 14                            |
|       | <u>6076</u>  |             | 86         | 16, <i>the whole profits.</i> |
|       | 3038         |             | <u>£37</u> | 2, <i>B's share.</i>          |
|       | <u>3038</u>  |             |            |                               |

|      |                |     |                |             |             |
|------|----------------|-----|----------------|-------------|-------------|
| (42) | 288793         | or, | 134786         | ·7,8,5,4)14 | (17·83      |
|      | <u>6874310</u> |     | <u>3978820</u> |             | <u>7854</u> |
|      | 28879          |     | 26957          |             | 6146        |
|      | 8664           |     | 10783          |             | <u>5498</u> |
|      | 1155           |     | 1078           |             | 648         |
|      | 202            |     | 94             |             | <u>628</u>  |
|      | 23             |     | 12             |             | 20          |
|      | <u>2</u>       |     | <u>038924</u>  |             |             |
|      | <u>038925</u>  |     |                |             |             |

(43) As three parts are of copper and one part of tin, the tin must be one-fourth of the whole  $\therefore \frac{150}{4}$  lbs. =  $37\frac{1}{2}$  lbs., the weight of tin  $\therefore 150$  lbs. —  $37\frac{1}{2}$  lbs. =  $112\frac{1}{2}$  lbs., the weight of copper, = 3 times the weight of tin.

|      |           |                     |                                                                |
|------|-----------|---------------------|----------------------------------------------------------------|
| (44) | 48        |                     |                                                                |
|      | <u>52</u> | <i>gals.</i>        | <i>gals.</i>                                                   |
| 100  | : 48      | :: 84               | : $40\frac{8}{25}$                                             |
|      |           | <u>48</u>           |                                                                |
|      |           | 672                 |                                                                |
|      |           | <u>336</u>          |                                                                |
|      |           | <i>pure spirit,</i> | $40\frac{32}{32}$ <i>gals.</i> = $40\frac{8}{25}$ <i>gals.</i> |
|      |           |                     | <i>the whole,</i> <u>84</u> <i>gals.</i>                       |
|      |           |                     | <i>water,</i> $43\frac{17}{25}$ <i>gals.</i>                   |

(45) The difference between the amount and principal is the interest: to obtain this interest, when principal time and rate are given, we multiply the product of the principal and number of years by the rate, and divide the result by 100. Consequently, when the interest is *given* we must multiply it by 100 to obtain the result just mentioned, and this divided by the product of the principal and number of years must obviously give the rate: therefore the work of the present Example is as follows:—

|            |                      |
|------------|----------------------|
| £          | £                    |
| 956        | 1314½                |
| <u>7½</u>  | <u>956</u>           |
| 6692       | 358½                 |
| <u>478</u> | <u>100</u>           |
| 7170       | ) 35850 (5 per cent. |
|            | <u>35850</u>         |

- (46)      3 lbs. tea = 7 lbs. coffee,  
           13 lbs. coffee = 48 lbs. sugar,  
           15 lbs. sugar = 28 lbs. soap,  
       How many lbs. soap = 6 lbs. tea?

$$\frac{7 \times 48 \times 28 \times 6}{3 \times 13 \times 15} = \frac{7 \times 16 \times 28 \times 2}{13 \times 5} = \frac{6272}{65} = 96\frac{32}{65}, \text{ the number of lbs. required.}$$

(47) The fractions reduced to a common denominator are  $\frac{40}{60}, \frac{20}{60}, \frac{15}{60}$  and  $\frac{12}{60}$ ; and the sum of them is  $\frac{77}{60}$ ; and as these numbers have the same proportion among themselves, as the numerators only, the denominators may be suppressed, and the proportions stated as follows:—

|                                                                       |                                                                                           |
|-----------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
| $77 : 30 :: \frac{21}{11} : 8 \frac{2}{11}$                           | $77 : 20 :: \frac{21}{11} : 5 \frac{5}{11}$                                               |
| $\frac{30}{11) 90}$                                                   | $\frac{20}{11) 60}$                                                                       |
| $\therefore 8s. 2\frac{2}{11}d.$<br><i>is</i><br>$A's \text{ share,}$ | $\therefore 5s. 5\frac{5}{11}d.$<br><i>is</i><br>$B's \text{ share.}$                     |
| $\frac{12}{11) 24}$<br>$2\frac{2}{11}d.$                              | $\frac{12}{11) 60}$<br>$5\frac{5}{11}d.$                                                  |
|                                                                       | $\therefore 4 \frac{1}{11} C's.$<br>$\frac{17 \frac{8}{11}}{21}$<br>$3 \frac{3}{11} D's.$ |

And as 15, the next number, is half of 30, the first number, C's share, must be half of A's, that is  $4s. 2\frac{1}{11}d.$ : B's share might also have been found by taking two-thirds of A's share. 2

*cwt. gr. lbs.*

(48)  $\begin{array}{r} 152 \quad 1 \quad 3 \text{ gross weight.} \\ \underline{10} \\ 1520 \text{ lbs.} \\ 2 \text{ lbs.} = \frac{1}{4} \text{ of } 10 \text{ lbs., (omitting oz.) for } 1 \text{ qr.} \\ \underline{1522 \text{ lbs. tare.}} \\ = 13 \text{ cwt. } 2 \text{ qrs. } 10 \text{ lbs} \end{array}$

$\begin{array}{r} 26 \overline{)138} \quad \quad 2 \quad \quad 21 \\ \underline{5} \quad \quad 1 \quad \quad 9 \text{ tret.} \\ 133 \quad \quad 1 \quad \quad 12 \text{ net weight.} \end{array}$

(49)  $\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \quad \text{£} \quad \text{s.} \quad \text{d.} \quad \text{£} \quad \text{s.} \quad \text{d.} \quad \text{£} \quad \text{s.} \quad \text{d.} \\ 2340 \quad 17 \quad 6 : 143 \quad 9 \quad 10 :: 137 \quad 10 \quad 8 : 8 \quad 8 \quad 7\frac{1}{4} \end{array}$

Or,  $\times$  by 6, 14045 5 : 860 19

Or 14045.25 : 860 95

137

602665

258285

86095

$\begin{array}{r} 10\text{s.} \quad \left| \begin{array}{l} \frac{1}{2} \\ \frac{1}{20} \\ \frac{1}{3} \end{array} \right| \quad 43048 \\ 6\text{d.} \quad \quad \quad \quad 2152 \\ 2\text{d.} \quad \quad \quad \quad 717 \end{array}$

140,45.25)118409.32 (£8.4306 = £8 8s. 7 $\frac{1}{4}$ d.

11236200 20

604732 8.612

561810 12

42922 7.344

42136 4

786 1.376

(50) 300 (See foot-note, p. 178, Arith.)

500  $\times$  2 = 1000

900  $\times$  3 = 2700

233,0)400,0(1 *mo. 2 we. 6 days*, the equated time.

233

167

4

668

466

202

7

1414

1398

16

(51) 
$$\begin{array}{rcl}
 & \text{£} & \\
 \text{May 1} & . & 50 \\
 \text{June 4} & . & 64 \times 34 = 2176 \\
 \text{Aug. 1} & . & 86 \times 92 = 7912 \\
 \text{Sept. 5} & . & 90 \times 127 = 11430 \\
 & & \hline
 & & 29,0 \quad ) \quad 2151,8 \text{ (74 days from May 1.} \\
 & & \hline
 & & 203 \quad \therefore \text{the time of pay-} \\
 & & \hline
 & & 21 \quad \text{ment is July 14th.} \\
 & & \hline
 & & 16 \\
 & & \hline
 & & 58 \\
 & & \hline
 \end{array}$$

*pay and  
time of each.*

(52) 
$$\begin{array}{rcl}
 6 & \times 6 \times & 6 = 216 \\
 2 & \times 4 \times & 12 = 96 \\
 1\frac{1}{2} & \times 3 \times & 110 = 495 \\
 & & \hline
 807 & : 36 & :: 4000 : \text{each officer's portion.} \\
 807 & : 8 & :: 4000 : \text{each midshipman's portion} \\
 807 & : 4\frac{1}{2} & :: 4000 : \text{each sailor's portion.}
 \end{array}$$

*Work of the first proportion.*

$$\begin{array}{r}
 36 \\
 \hline
 4000 \quad \text{£} \quad \text{s.} \quad \text{d.} \\
 807 \overline{) 144000} \quad ( 178 \quad 8 \quad 9\frac{1}{4} \text{ officer's portion.} \\
 \underline{807} \\
 6330 \\
 \underline{5649} \\
 6810 \\
 \underline{6456} \\
 354 \\
 \underline{20} \\
 7080 \\
 \underline{6456} \\
 624 \\
 \underline{12} \\
 7488 \\
 \underline{7263} \\
 225 \\
 \underline{4} \\
 900 \\
 \underline{807} \\
 93 \\
 \hline
 \end{array}$$

Now from looking at the above proportions, we see that 8 is  $\frac{1}{9}$  of

twice 36; therefore if twice an officer's portion be divided by 9, we shall obtain a midshipman's portion.

$$\begin{array}{r}
 \begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 178 \quad 8 \quad 9\frac{1}{4} \\
 \hline
 2 \\
 \hline
 9 \overline{) 356 \quad 17 \quad 6\frac{1}{2}} \\
 \underline{\text{£} 39 \quad 13 \quad 0\frac{3}{4}}
 \end{array}
 \end{array}
 \text{midshipman's portion.}$$

And it is further obvious, since 36 is 8 times  $4\frac{1}{2}$ , that if we divide an officer's portion by 8, we shall obtain a sailor's portion.

$$\begin{array}{r}
 \begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{d.} \\
 8 \overline{) 178 \quad 8 \quad 9\frac{1}{4}} \\
 \underline{\text{£} 22 \quad 6 \quad 1}
 \end{array}
 \end{array}
 \text{sailor's portion,}$$

the fraction of a farthing over being neglected.

The mode of proceeding adopted in the foregoing solution may be very often followed with advantage in the division of a quantity into *proportional parts*, since a single stating, to determine *one* of the parts, may frequently be made to suffice for the determination of *all*; as shown above, and also at pp. 125, 128, &c. And in this way the numerical operations usually performed in working the rule for proportional parts, may be considerably reduced.

---

\* \* It may be noticed here that the fraction at p. 79 may be further simplified by dividing numerator and denominator by 11.

THE END.

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